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There's A Geometric Structure Hidden Inside The Brains Of Rats

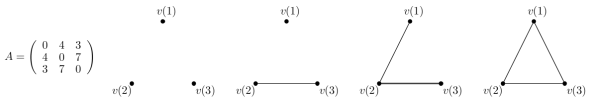
A rat looks for explosives inside a labyrinth. MAURICIO DUENAS/AFP/Getty Images.

How does a rat know its position in a maze? The key lies in the hippocampus, where particular neurons called *place cells* communicate with each other to help the animal sense where it is. A standard experiment in neuroscience is to record the activity of a collection of place cells while the rat explores its environment and then search for correlations between neuron firings and the animal's location.

In a recent paper in the *Proceedings of the National Academy of Sciences*, an interdisciplinary team of researchers used techniques from computational topology to discover intrinsic geometric structure in this neural activity data. Such data is usually presented as a matrix $C(i,j)$ where the k,l entry indicates the strength of connectivity between neuron k and neuron l. It could be that there is no particular structure to this matrix; that is, the entries may be effectively random. Another possibility is that structure exists in the matrix because of the particular coding of the neurons rather than their actual location in the

hippocampus. Place cells fire rapidly when the rat is in their preferred region of the environment, so it is desirable to have a method to detect the geometric organization of the network using only intrinsic properties of the matrix $C(i,j)$.

The new idea is to analyze the *order complex* of the matrix using *persistent homology*. Say we are dealing with an $n \times n$ matrix $A(i,j)$. The order complex is built by taking a vertex $v(j)$ for $j=1,\ldots n$ and then adding an edge from $v(k)$ to $v(l)$ in decreasing order of the entries of the matrix; this is really a nested sequence of graphs. Since the neural data matrix is symmetric (the i,j entry equals the j,i entry), we consider matrices of this type only. Here is a simple example in the case $n=3$.

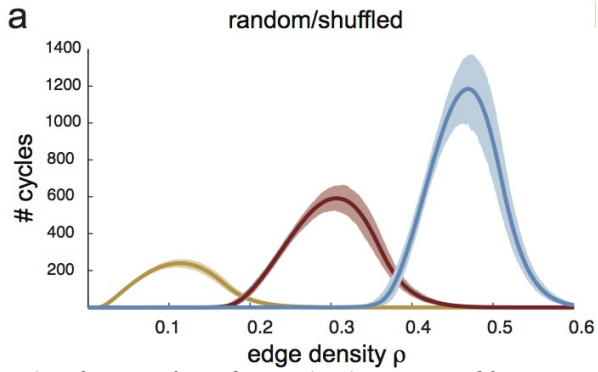


A *clique* in a graph is a subgraph where each vertex is connected to all the others. The triangle above has several cliques: the triangle itself and each edge. If we have a clique, we can imagine filling it in, and then collections of cliques might bound something that can't be filled in. For example, if four vertices form a square (in which the diagonal vertices are not connected), then each edge is a clique and the four cliques gang up to make a loop that isn't filled in (we call this a *1-cycle*). Or, say we had a tetrahedron consisting of four triangular faces. Then each of the triangles is a clique and the four of them bound a 3-dimensional void that isn't filled in (we then call the tetrahedron a *2-cycle*). Homology is the algebraic mechanism to encode all the cycles in a complex, and it can be computed via standard linear algebra. The kth Betti number is the count of the number of independent k-cycles.

Now here's the idea. Take a symmetric matrix and look at its order complex. There will be many cliques, but we want to look at them in a clever way. As we build the order complex by adding edges, we can think of this as a corresponding adjacency matrix–the i,j and j,i entries are 1 if the edge exists in the order complex and 0 otherwise. We may then measure the density $\rho = k/[n(n+1)/2]$ of nonzero entries in the

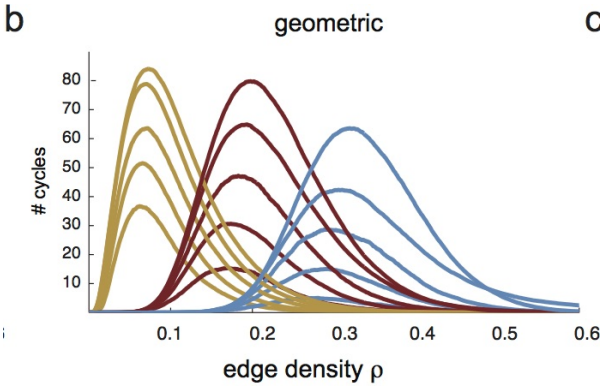
adjacency matrix (*k* is the number of edges in the *k*th stage of the order complex) and watch how cliques evolve as we increase ρ from 0 upwards. In the picture above, the densities are 0, 1/3, 2/3, and 1, respectively. Cliques may form cycles of various dimensions at one value of ρ but then these cycles may get filled in at higher density values as more edges (and therefore more triangles, solid tetrahedra, etc.) appear. Keeping count of these as ρ increases measures the persistent homology of the order complex (cycles *persist* for a range of density values, hence the name) and we may graph the Betti numbers against density.

Suppose we take a random symmetric matrix. What should we expect these curves of Betti numbers to look like? The authors ran experiments and discovered that the graphs look like this:



Betti number curves for random matrices, image excerpted from original paper.

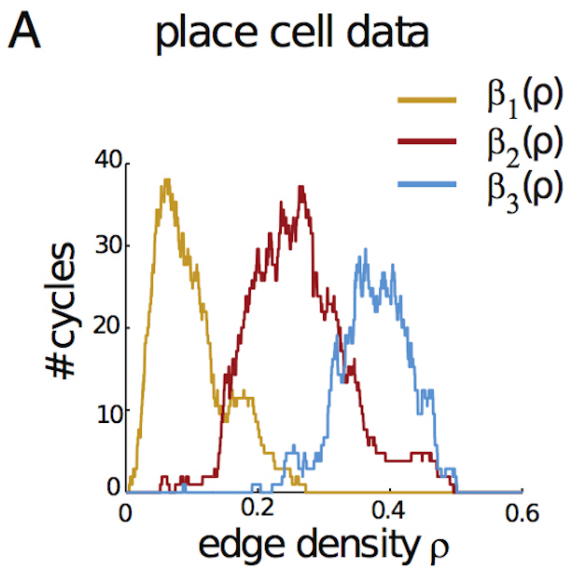
On the other hand, for matrices built from pairwise distances in space (a so-called *geometric matrix*), the Betti curves look like:



Betti curves associated to geometric data, image excerpted from original paper.

There are clear differences in these signatures, suggesting that this technique is able to differentiate random data from geometric. Returning to the rodent neural activity data,

where the rat is exploring its 2-dimensional environment, the analysis gives this Betti number profile:



Betti numbers associated to neural activity, image excerpted from original paper.

Clearly, this resembles the profile associated to geometric data rather than random data. This is the first time geometric structure has been found intrinsically in neural data. Certainly such a structure is to be expected since the rat's place cells keep track of the geometry of the environment, but this result is confirmation that it can be detected using only the pattern of correlations among the neurons. And it suggests that such geometric structure is a property of the underlying place cell network and not a result of the spatial structure of the input cells.

In the end, there is still a great deal that we do not know about the structure of rat brains, never mind those of humans, but any new technique to understand how they work is welcome. We can add the study of clique topology to the toolbox.