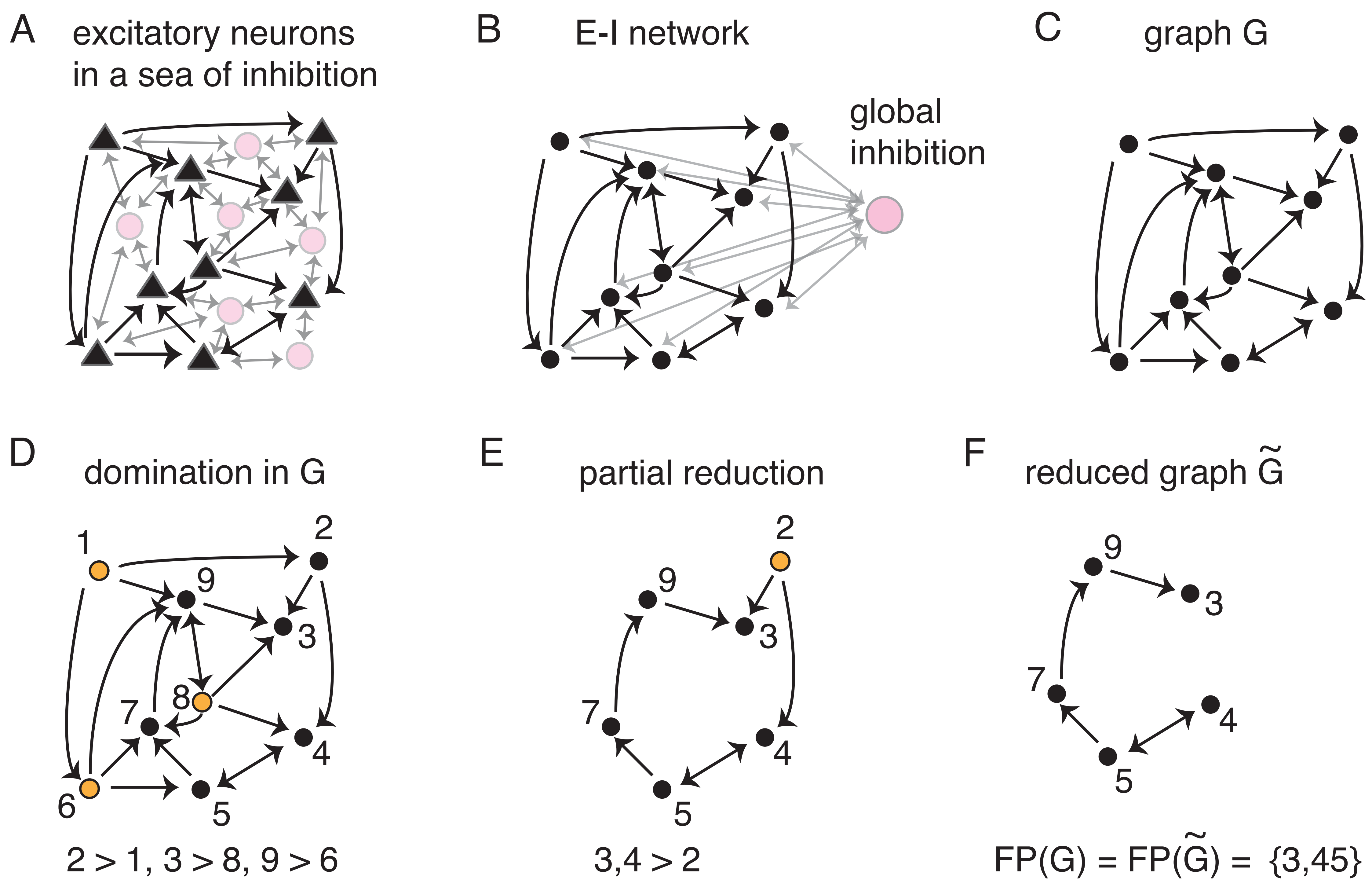
Domination and reduction for E-I TLNs built from graphs



Carina Curto, Brown University

Janelia workshop: Analysis and Modeling of Connectomes

June 3, 2025

Motivating questions and ideas:

1. How does connectivity shape dynamics?

2. The relationship between connectivity and neural activity depends on the dynamical system you associate to the connectome.

3. By studying neuroscience-inspired (nonlinear!) dynamical systems on graphs, we can generate hypotheses about the dynamic meaning/role of various network motifs.

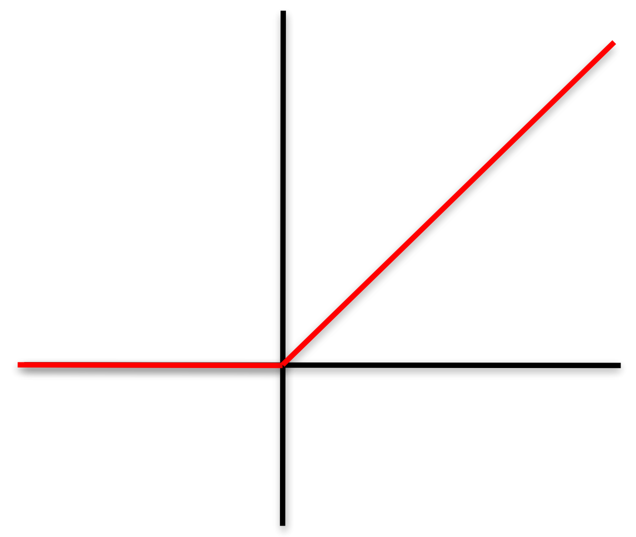
TLNs — nonlinear recurrent network models

Threshold-linear network dynamics:

$$\frac{dx_i}{dt} = -x_i + \left[\sum_{j=1}^{n} W_{ij}x_j + b_i\right]_+$$

W is an $n \times n$ matrix

$b \in \mathbb{R}^n$



The TLN is defined by (W, b)

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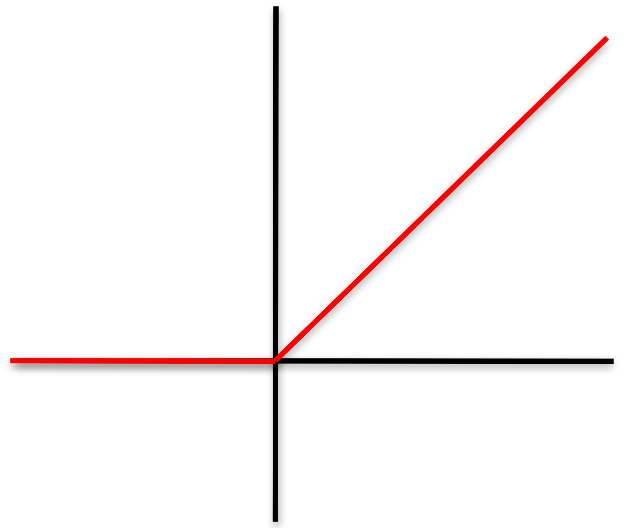

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Basic Question: Given (W,b), what are the network dynamics?

TLNs — nonlinear recurrent network models

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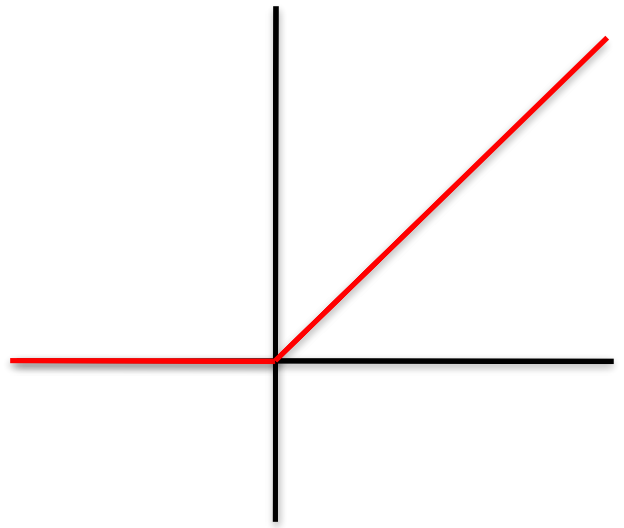


The TLN is defined by (W, b)

Linear network dynamics:

$$\frac{dx}{dt} = Ax + b$$

A is an $n \times n$ matrix

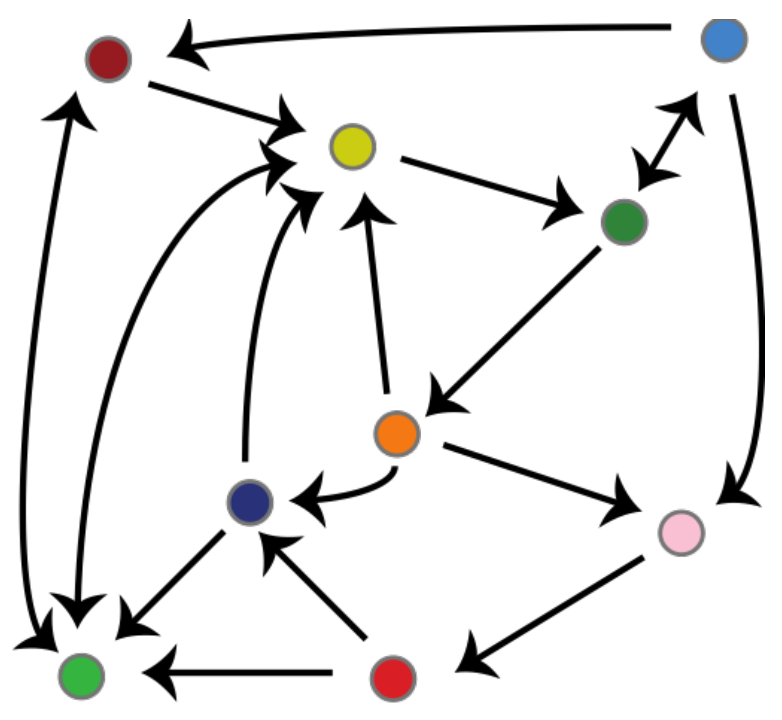
$b \in \mathbb{R}^n$

Long-term behavior is easy to infer from eigenvalues, eigenvectors — linear algebra tells us everything.

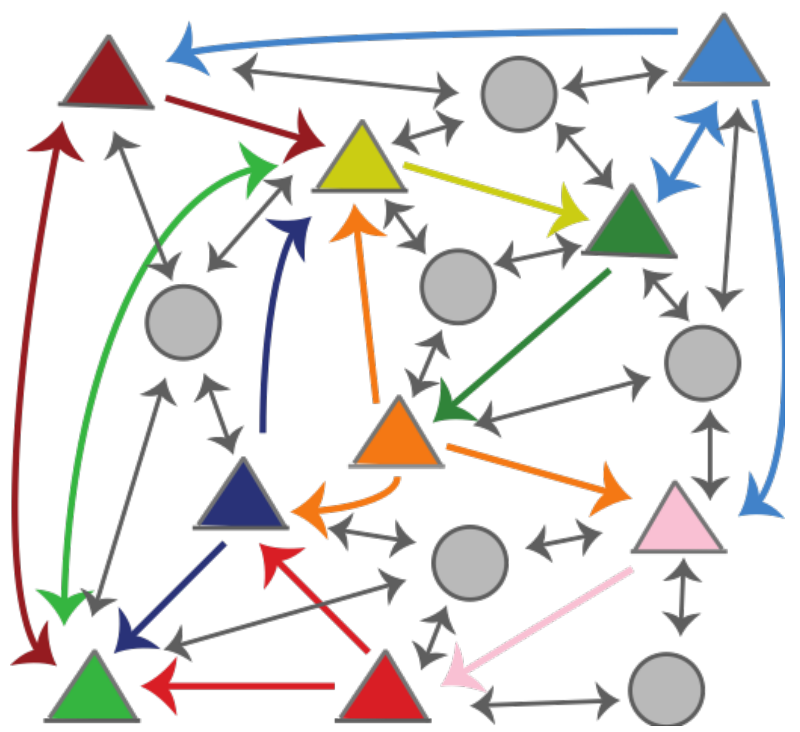
Basic Question: Given (W,b), what are the network dynamics?

The most special case: Combinatorial Threshold-Linear Networks (CTLNs)

graph G



Idea: network of excitatory
and inhibitory cells



TLN dynamics:

$$\frac{dx_i}{dt} = -x_i + \left[\sum_{j=1}^{n} W_{ij}x_j + \theta\right]_+$$

Graph G determines the matrix W

$$W_{ij} = \begin{cases} 0 & \text{if } i = j \\ -1 + \varepsilon & \text{if } i \leftarrow j \text{ in } G \\ -1 - \delta & \text{if } i \not\leftarrow j \text{ in } G \end{cases}$$

parameter constraints:

$$\delta > 0 \qquad \theta > 0 \qquad 0 < \varepsilon < \frac{\delta}{\delta + 1}$$

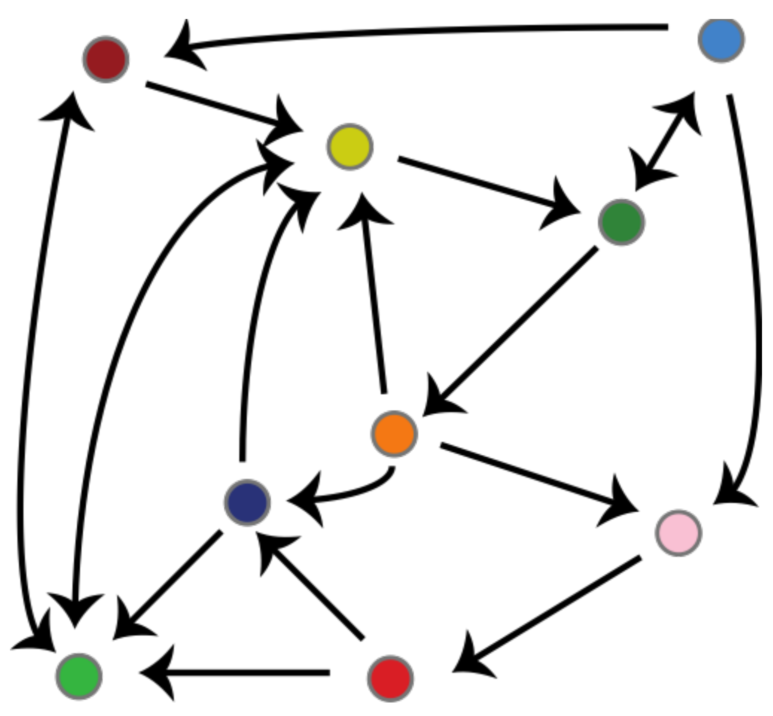
The graph encodes the pattern
of weak and strong inhibition

Think: generalized WTA networks

For fixed parameters,
only the graph changes –
isolates the role of connectivity

Less special: generalized Combinatorial Threshold-Linear Networks (gCTLNs)

graph G



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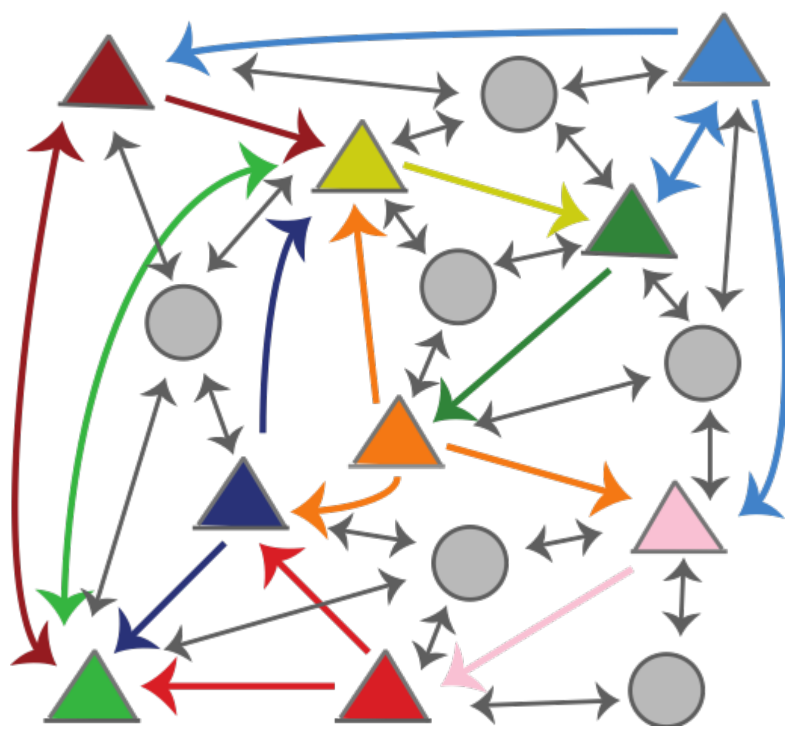


TLN dynamics:

$$\frac{dx_i}{dt} = -x_i + \left[\sum_{j=1}^{n} W_{ij}x_j + \theta\right]_+$$

The gCTLN is defined by a graph G and two vectors of parameters: ε, δ

$$W_{ij} = \begin{cases} -1+\varepsilon_j & \text{if } j \to i, \quad \text{weak inhibition} \\ -1-\delta_j & \text{if } j \not\to i, \quad \text{strong inhibition} \\ 0 & \text{if } i = j. \end{cases}$$

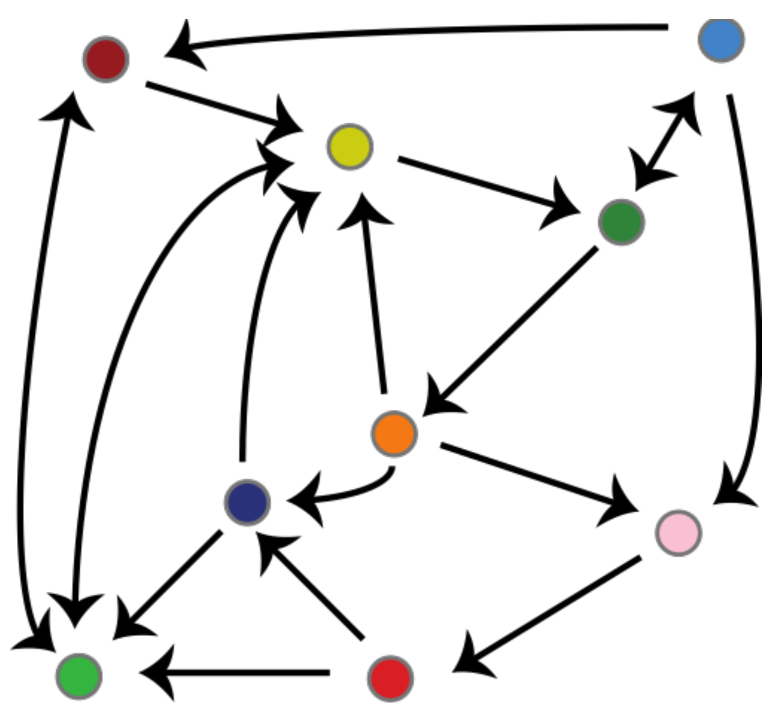
The graph encodes the pattern of weak and strong inhibition

$b_i = \theta > 0$ for all neurons

(default is uniform across neurons, constant in time)

Less special: generalized Combinatorial Threshold-Linear Networks (gCTLNs)

graph G



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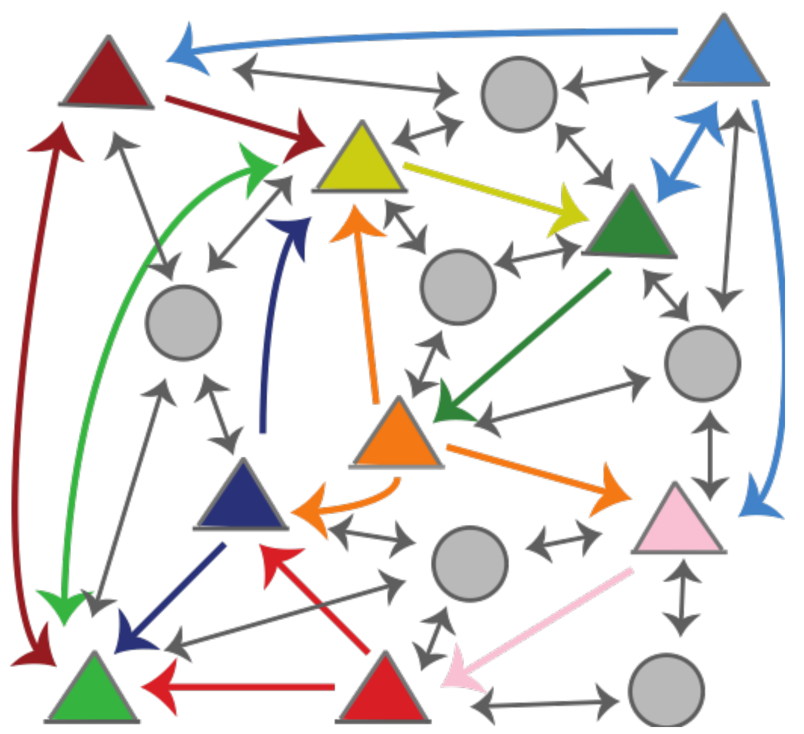

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weak inhibition (if $j \to i$)

strong inhibition (if $j \not\to i$)

The graph encodes the pattern of weak and strong inhibition

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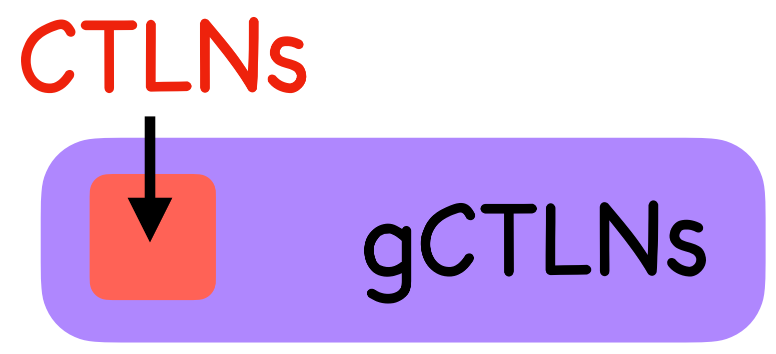
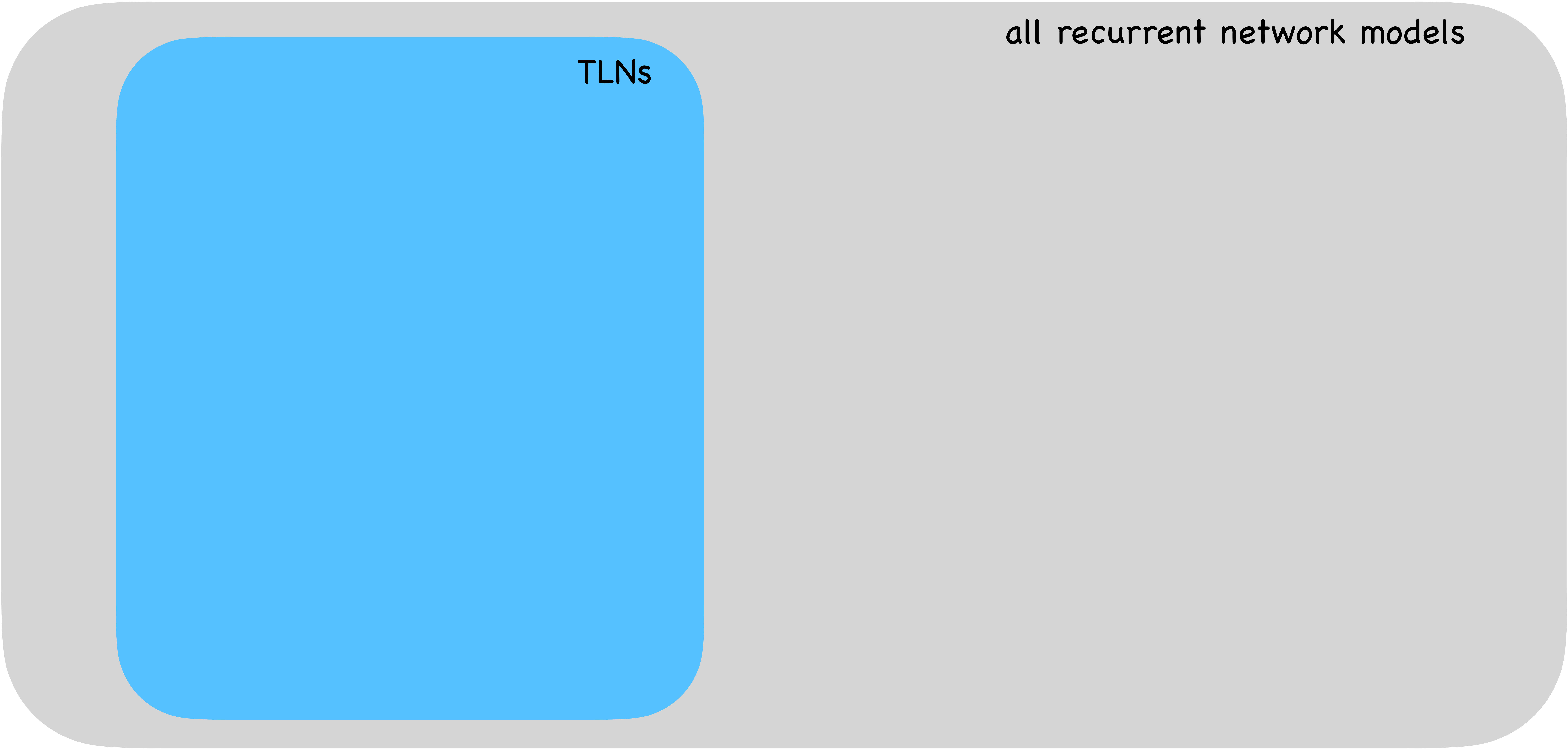
(default is uniform across neurons, constant in time)



Special case: if the parameters ε_j, δ_j are the same for all neurons, we have a CTLN.

TLNs, CTLNs, and gCTLNs



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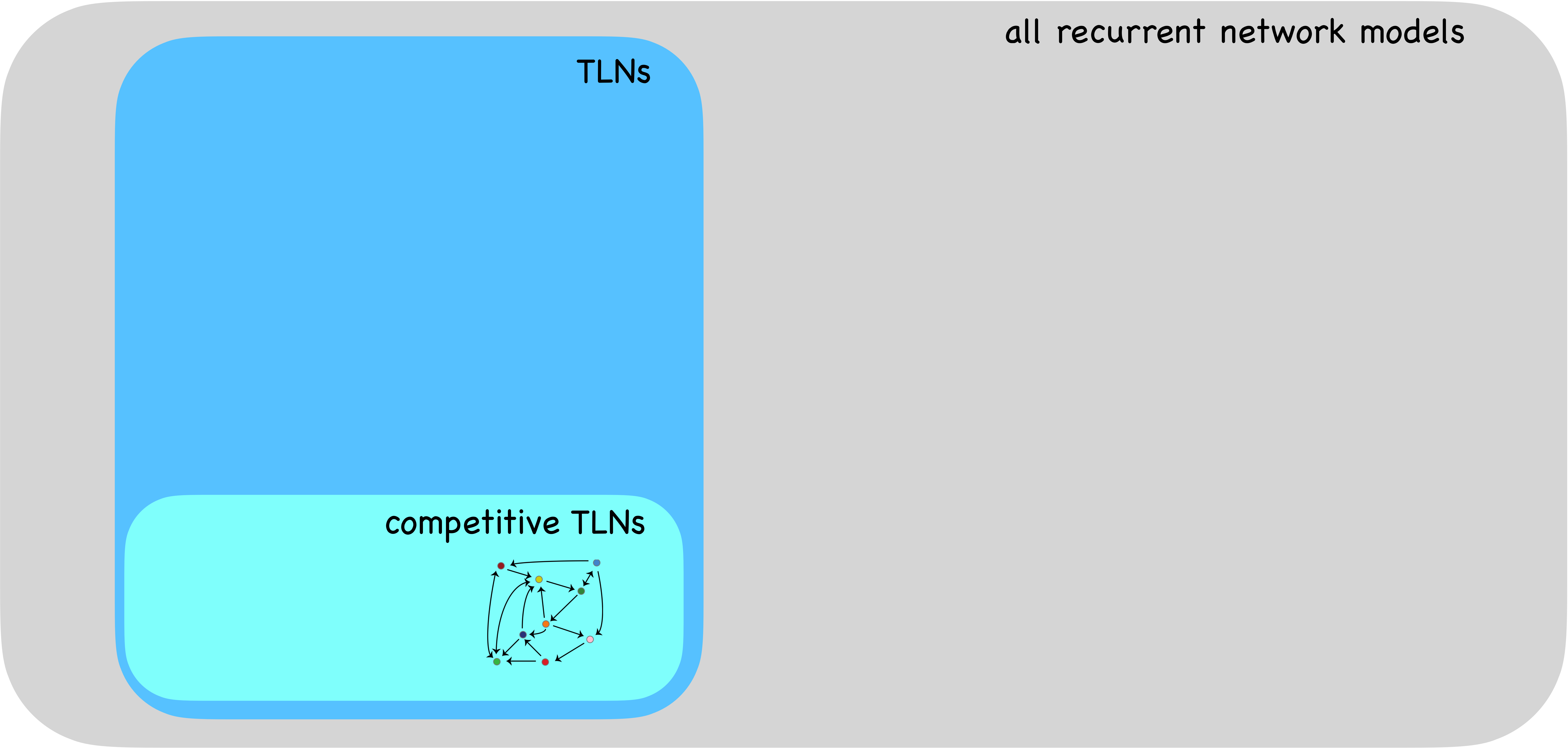

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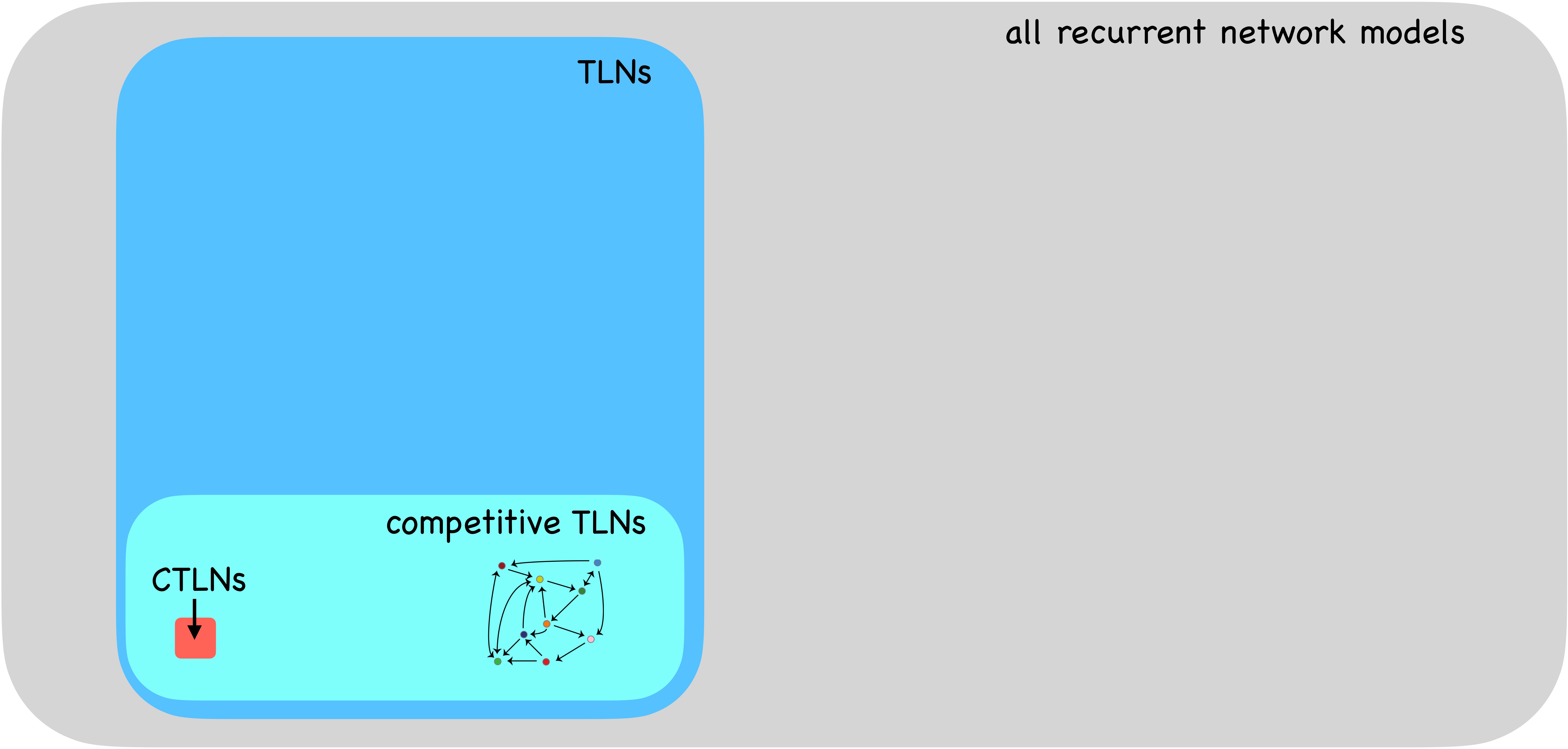

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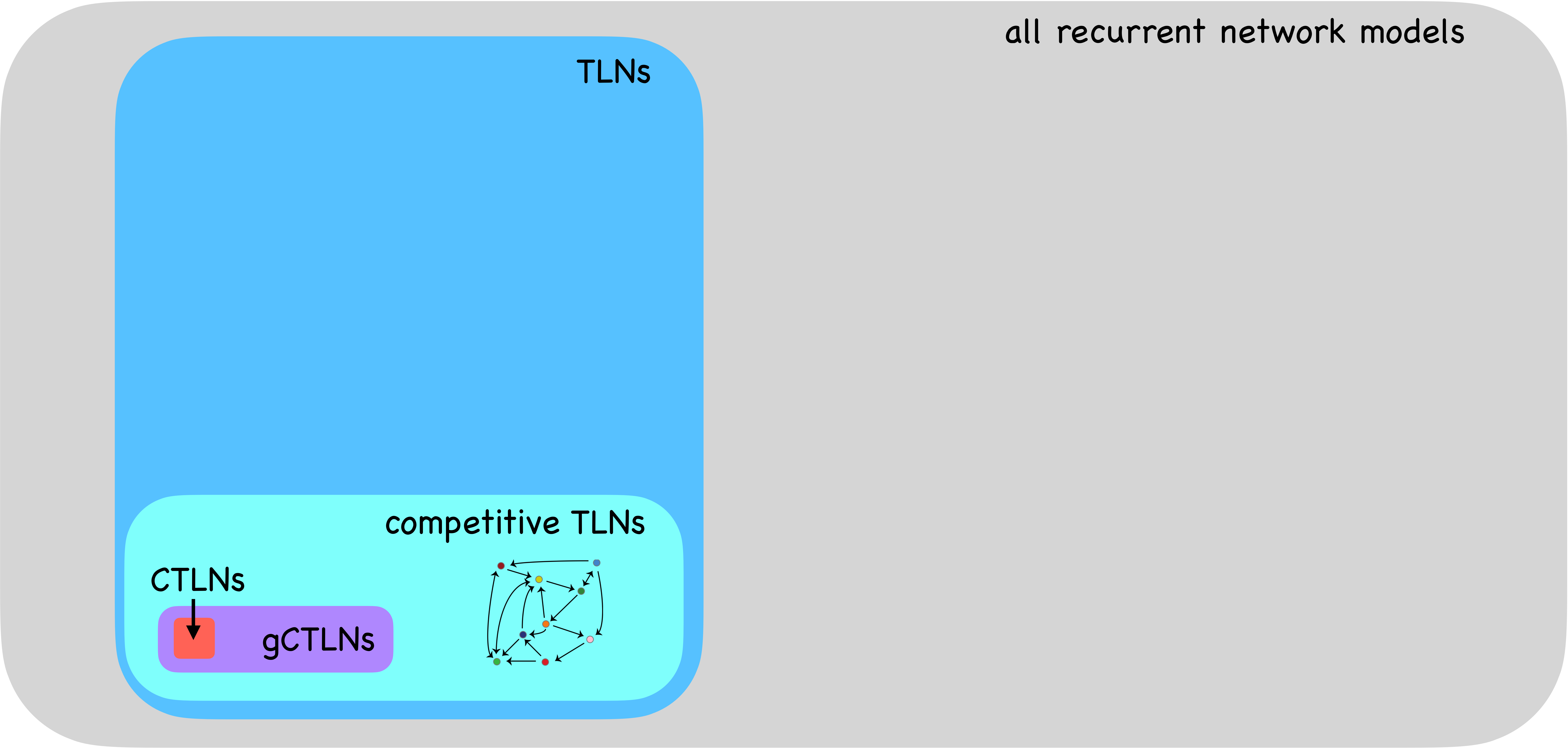

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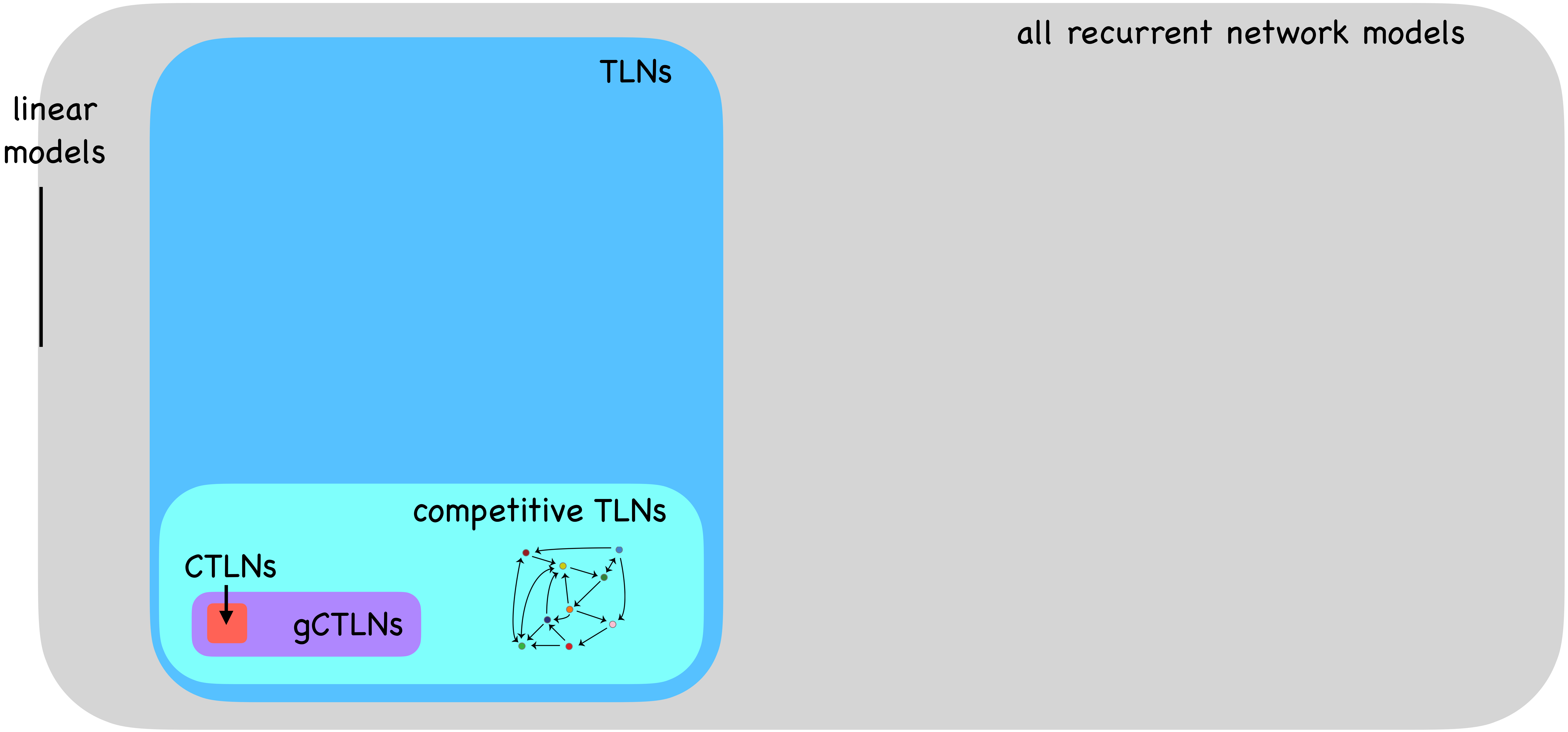
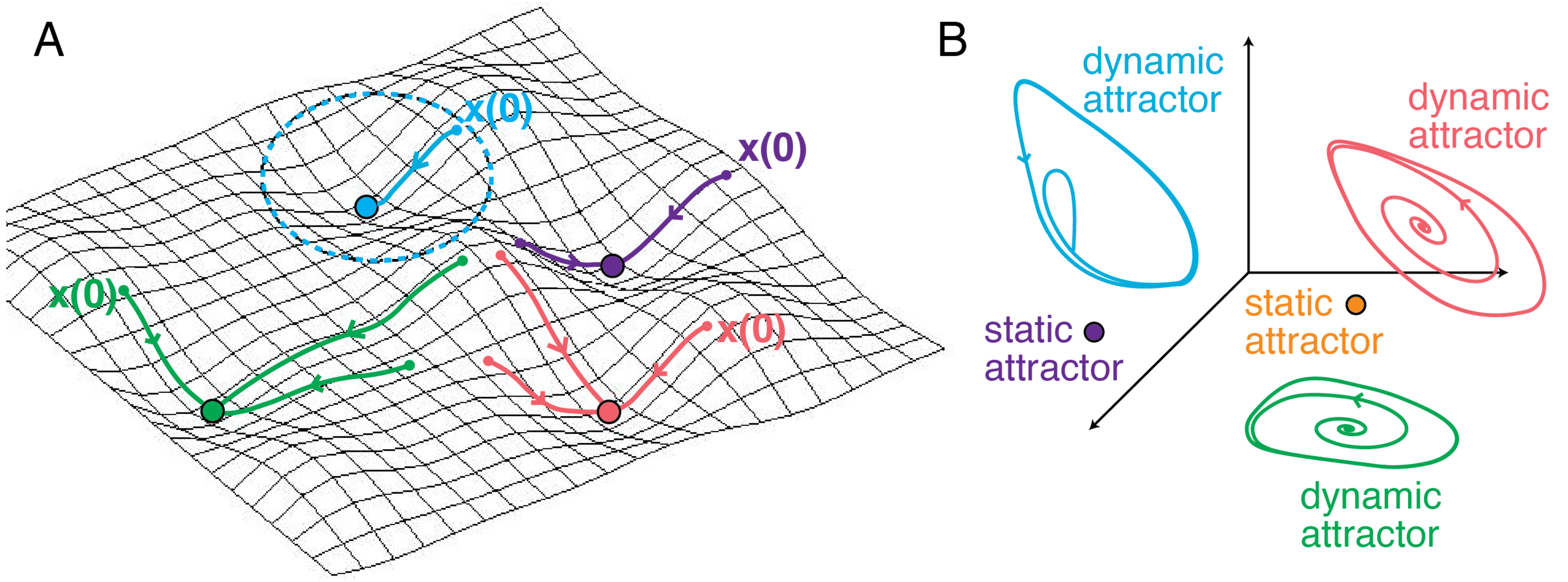


TLNs, CTLNs, and gCTLNs

1. Display rich nonlinear dynamics: multistability, limit cycles, chaos…

static attractors (fixed pts)

dynamic attractors
(correspond to certain unstable fixed pts)



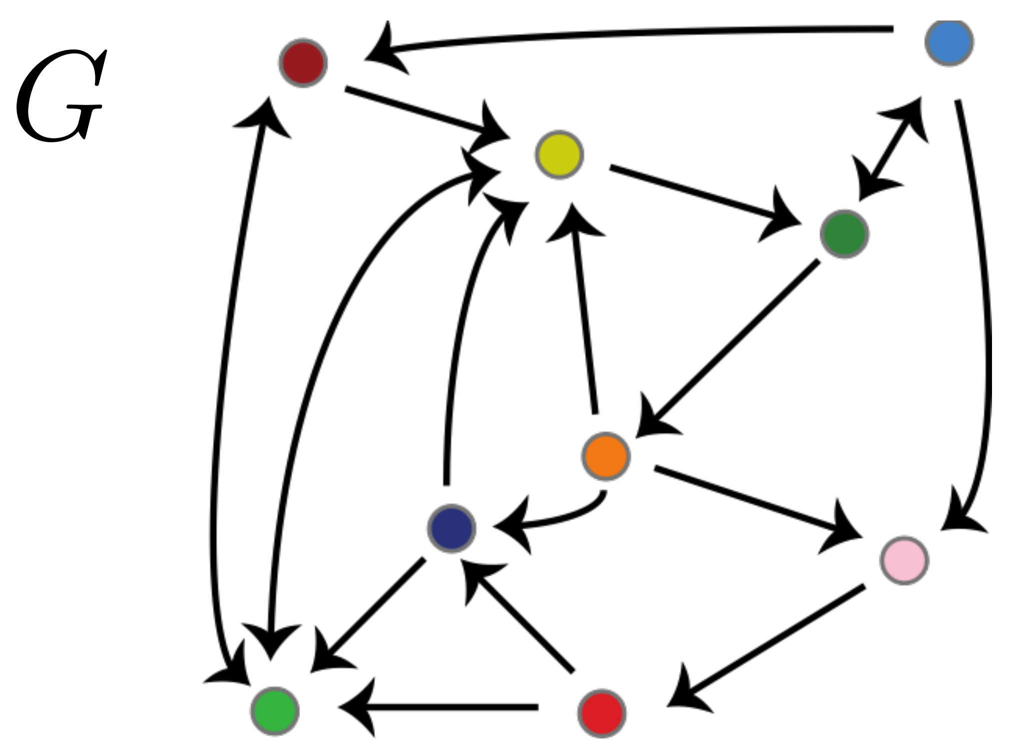
Curto & Morrison, 2023 (review paper): Graph rules for recurrent neural network dynamics

TLNs, CTLNs, and gCTLNs

1. Display rich nonlinear dynamics: multistability, limit cycles, chaos…

2. Mathematically tractable: we can prove theorems directly connecting graph structure to dynamics.

graph structure



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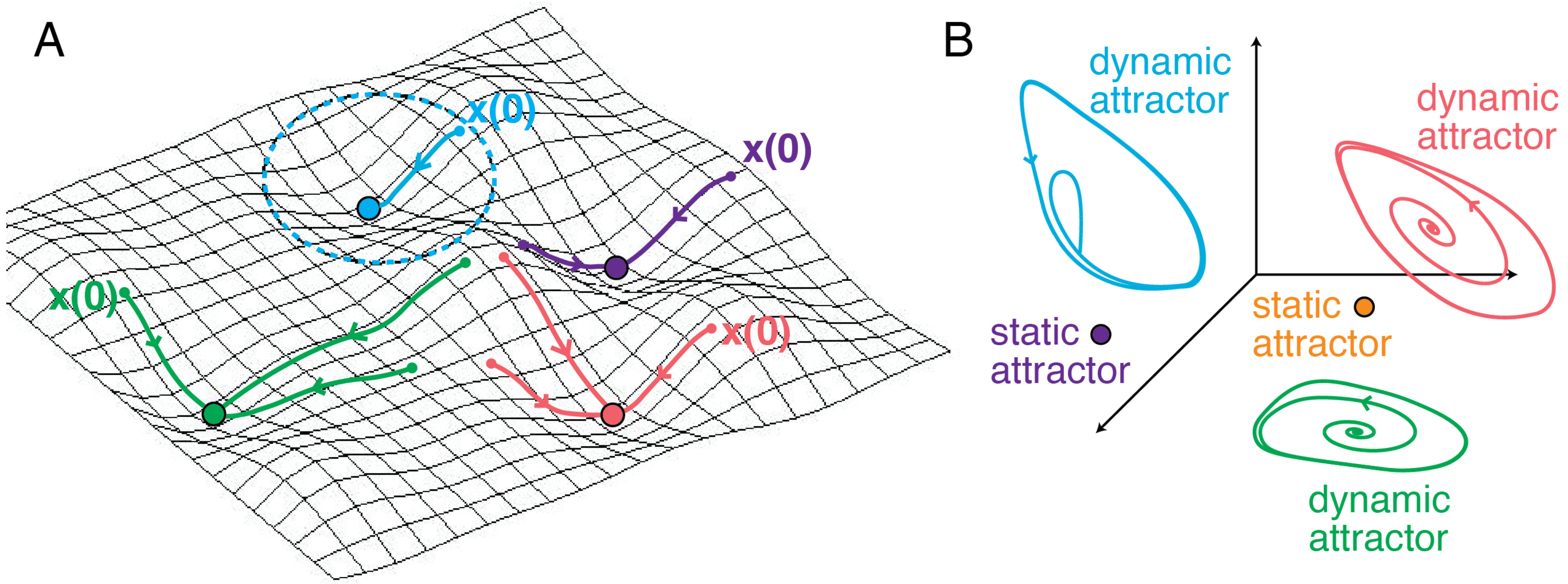

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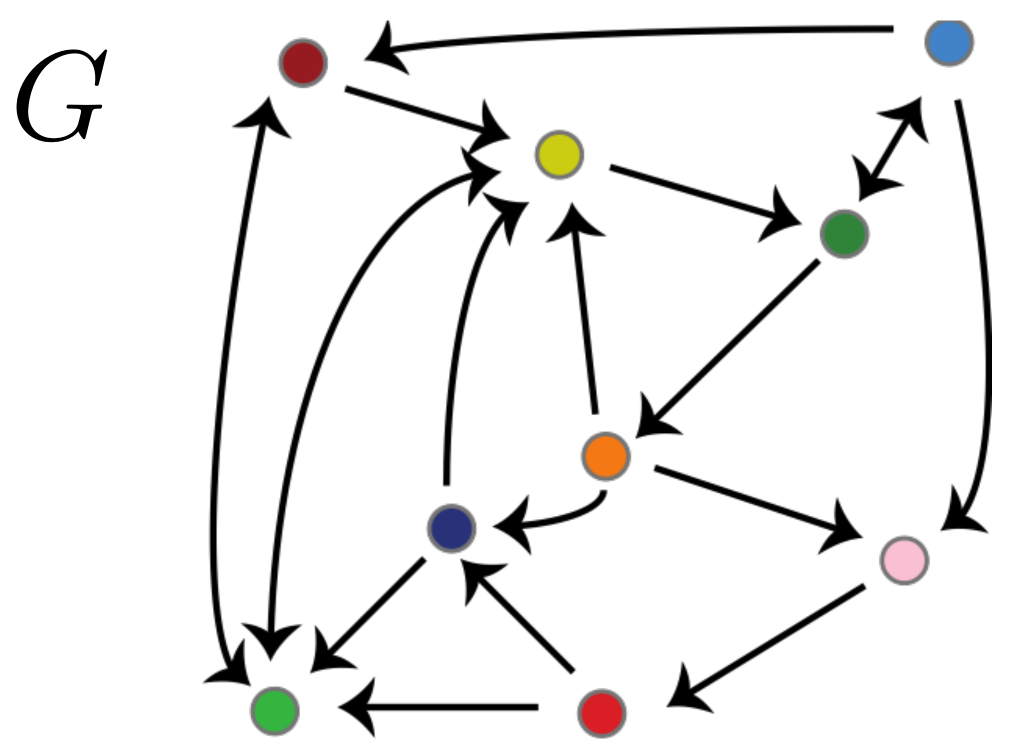
TLNs, CTLNs, and gCTLNs

1. Display rich nonlinear dynamics: multistability, limit cycles, chaos...

2. Mathematically tractable: we can prove theorems directly connecting graph structure to dynamics.

3. Both stable and unstable fixed points play a critical role in shaping the dynamics (the vector field).

graph structure



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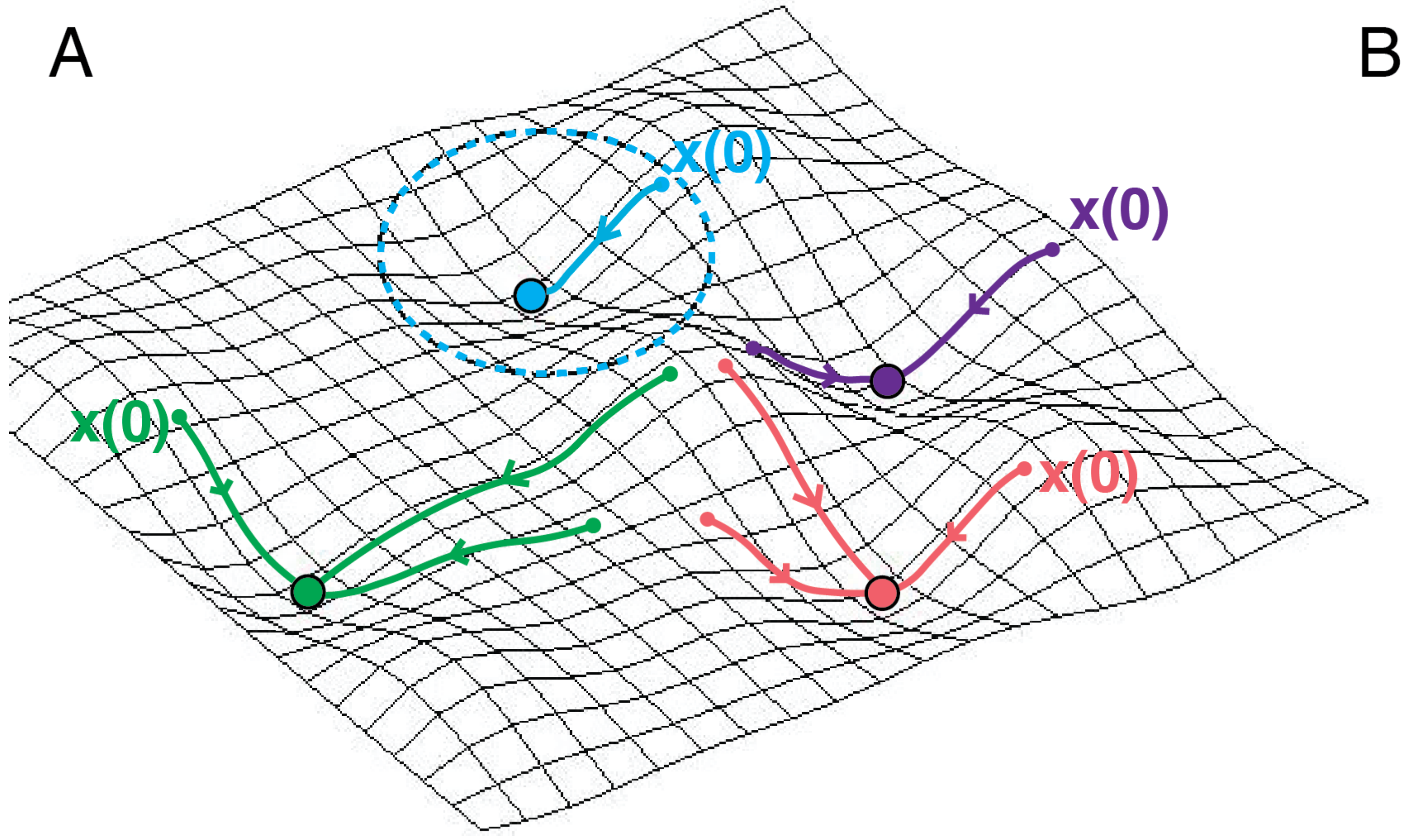

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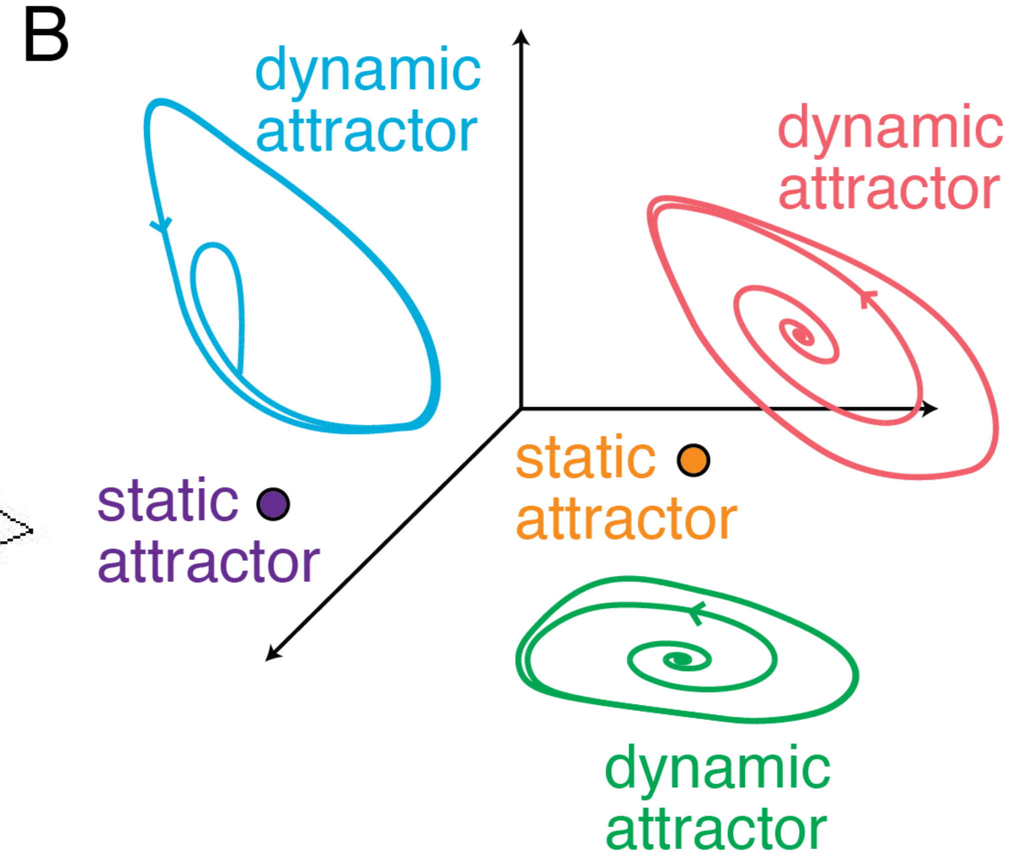
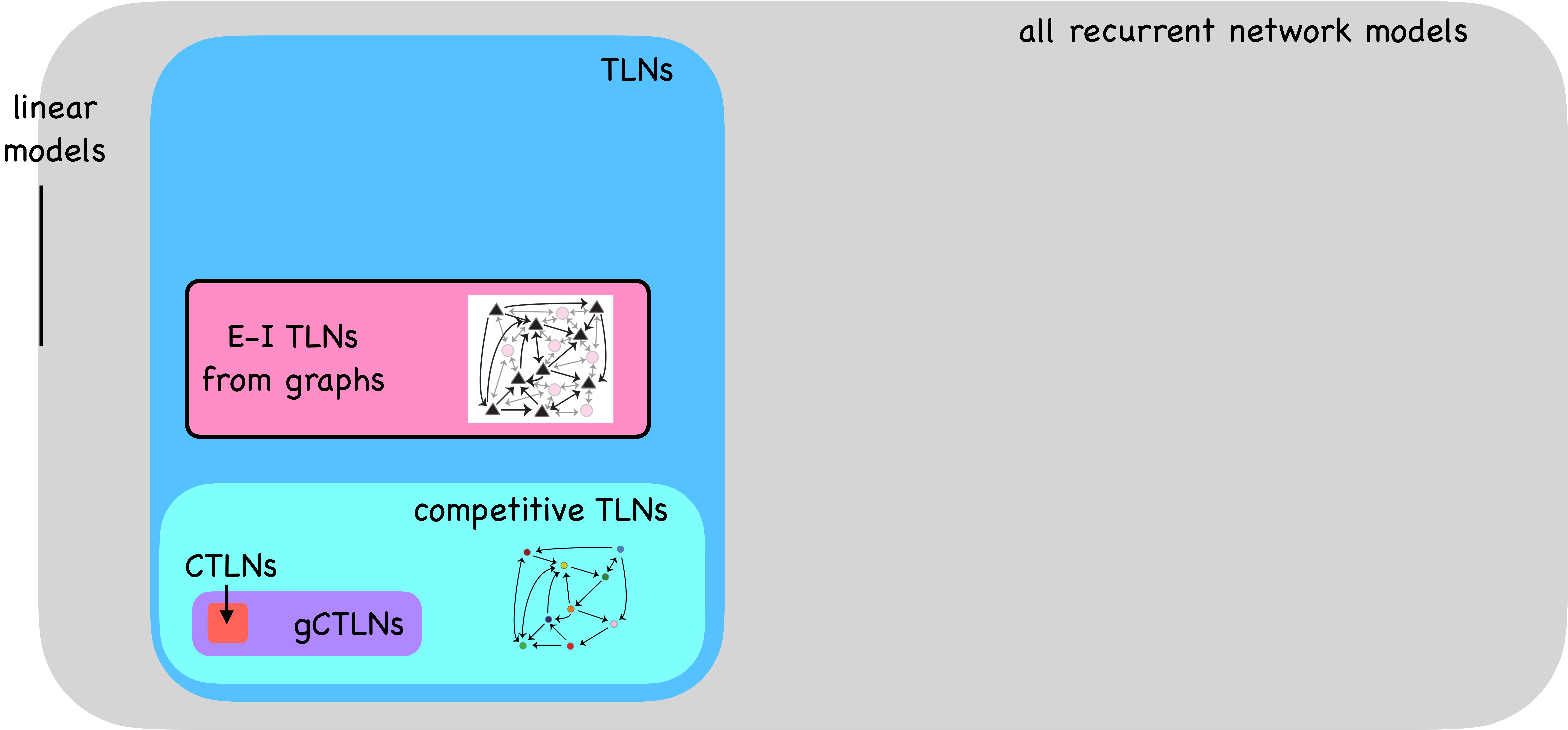


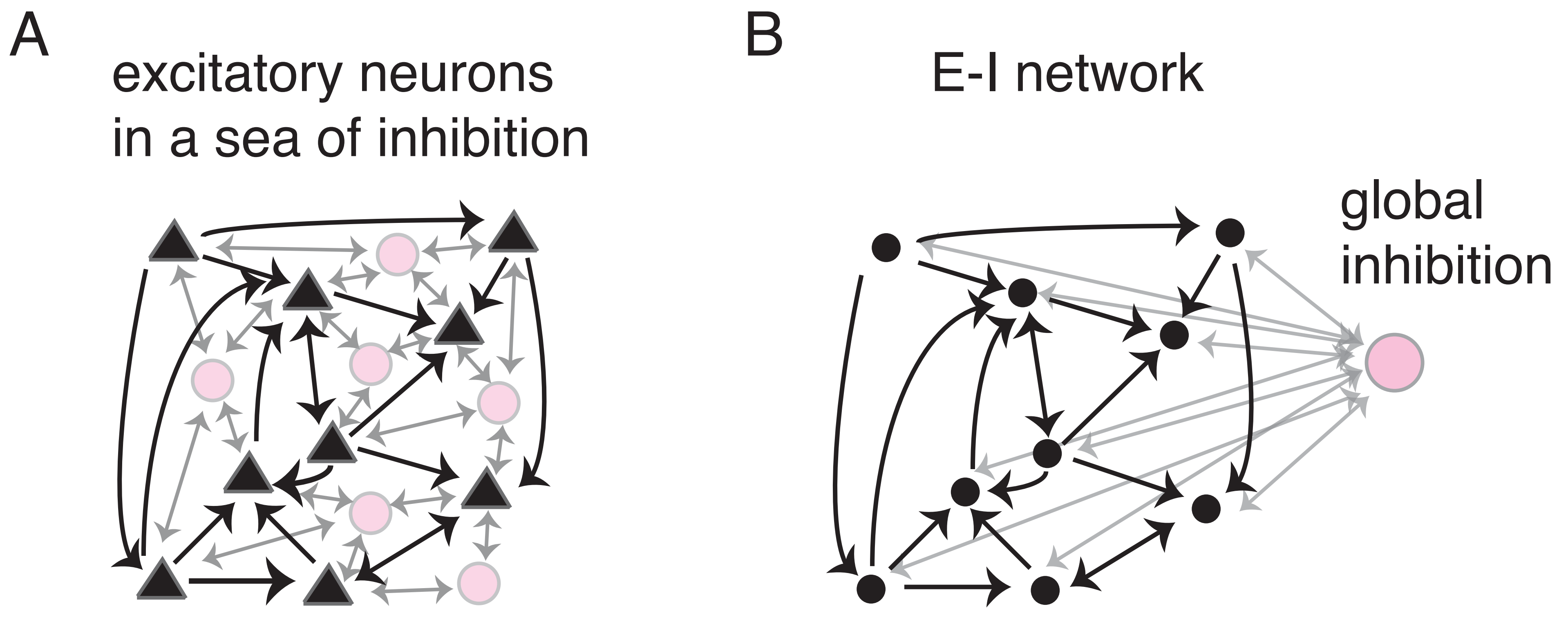
$$\mathrm{FP}(G) = \mathrm{FP}(G, \varepsilon, \delta) = \{ \text{ fixed points (stable and unstable) } \}$$

Curto & Morrison, 2023 (review paper): Graph rules for recurrent neural network dynamics

TLNs, CTLNs, and gCTLNs … and E-I TLNs from graphs



E-I TLNs from graphs



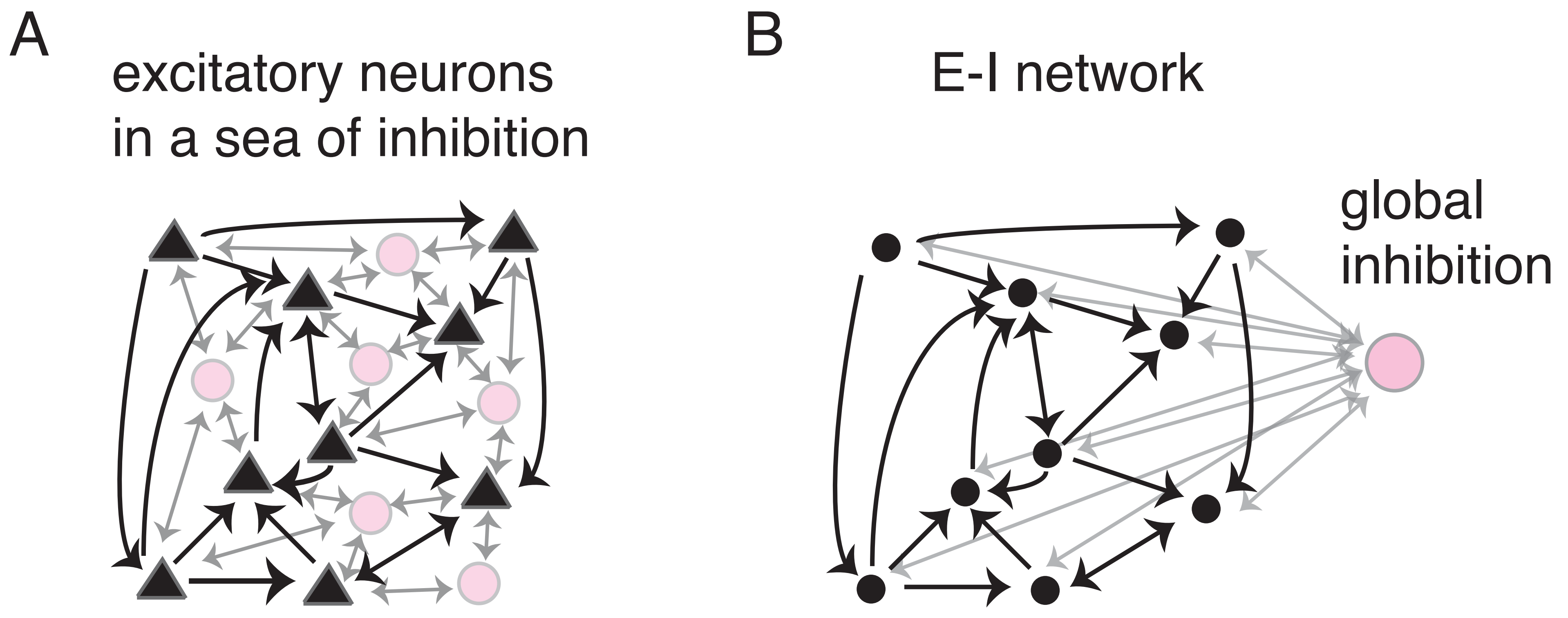
$$\frac{dx_i}{dt} = -x_i + \left[\sum_{j=1}^{n} W_{ij}x_j + W_{iI}(x_I - W_{Ii}x_i) + b_i\right]_+, \; i = 1, \ldots, n,$$

$$\frac{dx_I}{dt} = \frac{1}{\tau_I}\left(-x_I + \left[\sum_{j=1}^{n} W_{Ij}x_j + b_I\right]_+\right).$$

$$W_{ij} = \begin{cases} a_j & \text{if } j \to i \text{ in } G, \\ 0 & \text{if } j \not\to i \text{ in } G, \\ 0 & \text{if } i = j, \end{cases} \quad \text{and} \quad \begin{aligned} W_{Ij} &= c_j, \\ W_{iI} &= -1, \\ W_{II} &= 0. \end{aligned}$$

Curto 2025 (preprint soon!)

E-I TLNs from graphs



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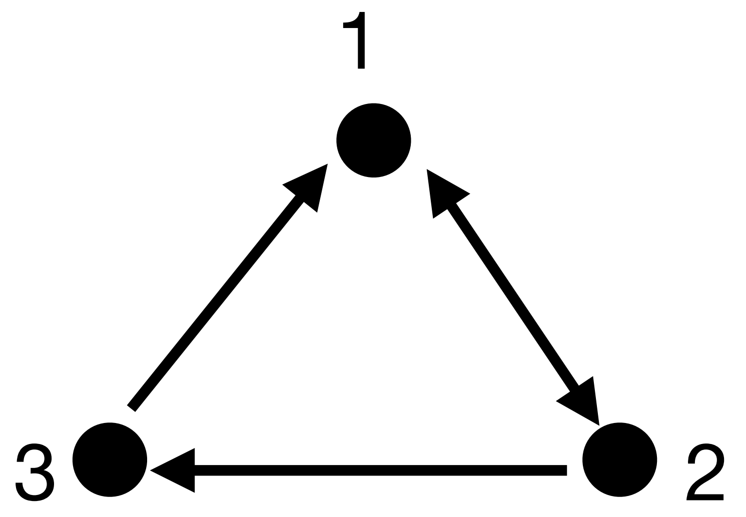
Example G:



W for E-I TLN

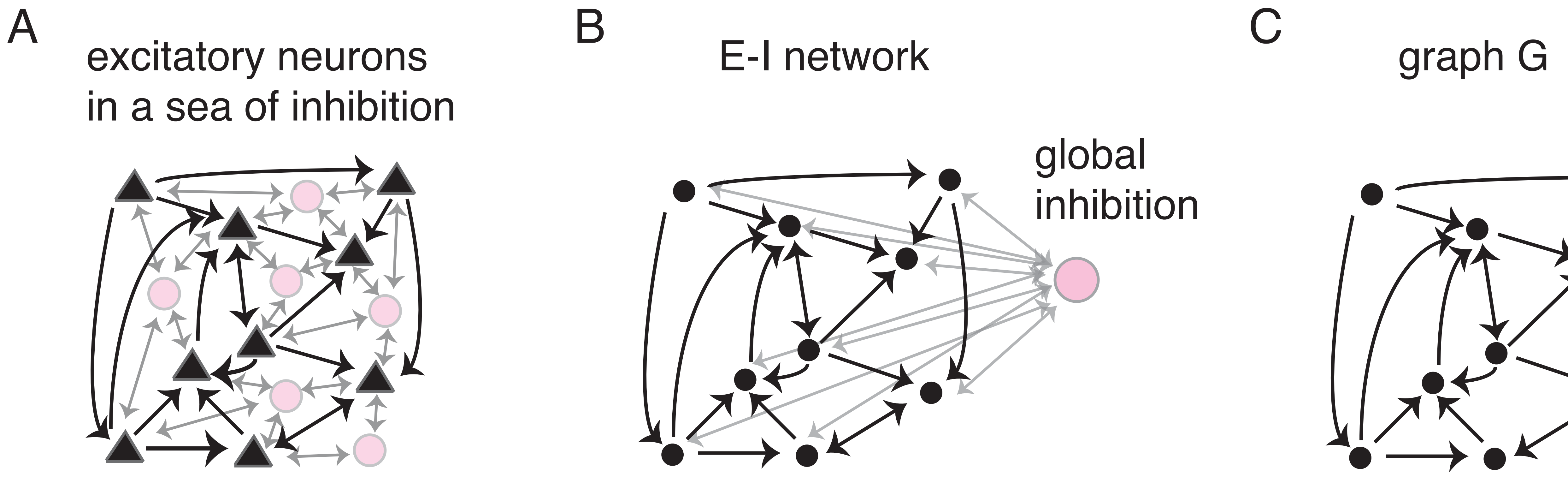
$$W = \begin{pmatrix} 0 & a_2 & a_3 & -1 \\ a_1 & 0 & 0 & -1 \\ 0 & a_2 & 0 & -1 \\ c_1 & c_2 & c_3 & 0 \end{pmatrix}$$

W for gCTLN

$$W = \begin{pmatrix} 0 & -1+\varepsilon_2 & -1+\varepsilon_3 \\ -1+\varepsilon_1 & 0 & -1-\delta_3 \\ -1-\delta_1 & -1+\varepsilon_2 & 0 \end{pmatrix}$$

Curto 2025 (preprint soon!)

There is a mapping from E-I TLNs to gCTLNs that preserves fixed points



inhibitory interaction

$$\frac{dx_i}{dt} = -x_i + \left[\sum_{j=1}^{n} W_{ij}x_j + W_{iI}(x_I - W_{Ii}x_i) + b_i\right]_+, \quad i = 1, \dots, n,$$

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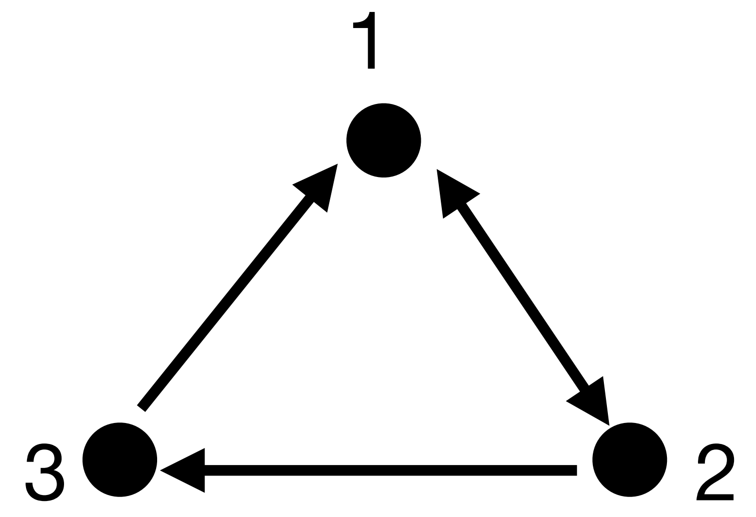
Parameter mapping to get the same fixed points:

$$\varepsilon_j = 1 + a_j - c_j,$$
$$\delta_j = c_j - 1.$$

Example G:



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C. Lienkaemper, G. Ocker. Dynamics of clustered spiking networks via the CTLN model (2025)

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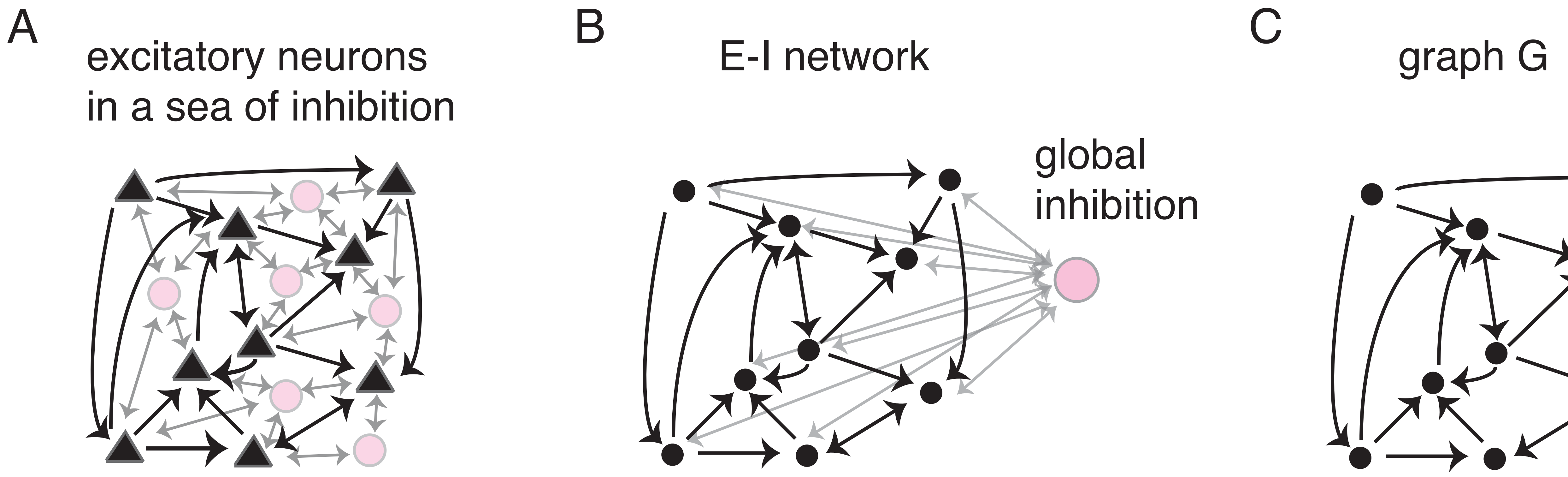

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Parameter mapping to get the same fixed points:

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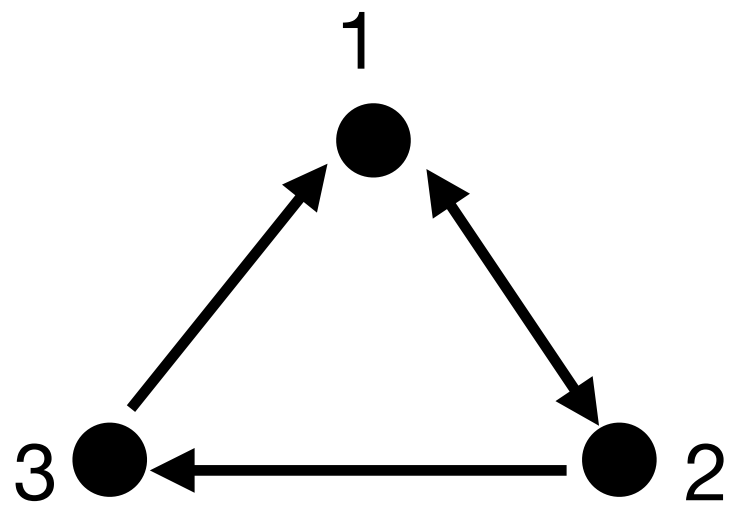
The mapping says nothing about the timescale of inhibition!

Example G:



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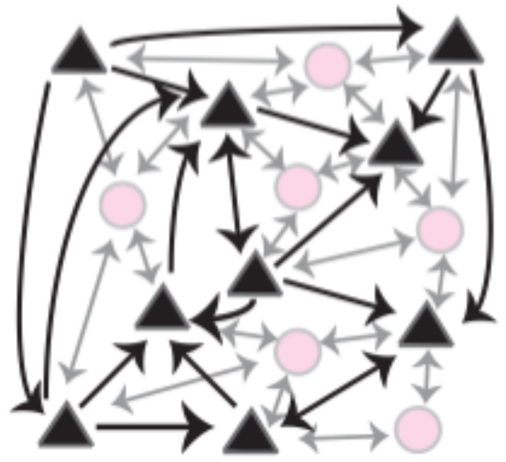
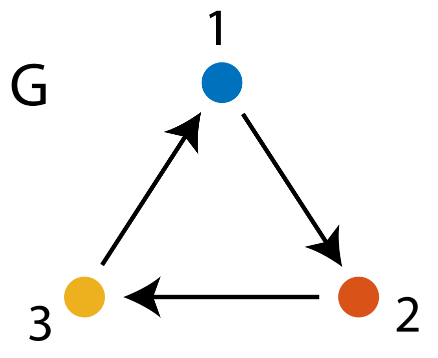
Beyond fixed points: do E-I TLNs produce similar dynamics to gCTLNs?

excitatory neurons
in a sea of inhibition



A



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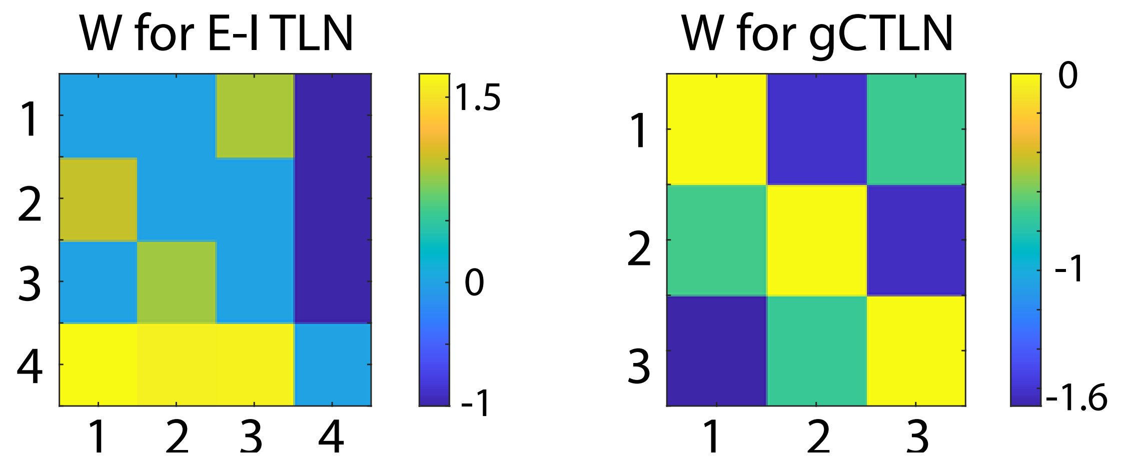

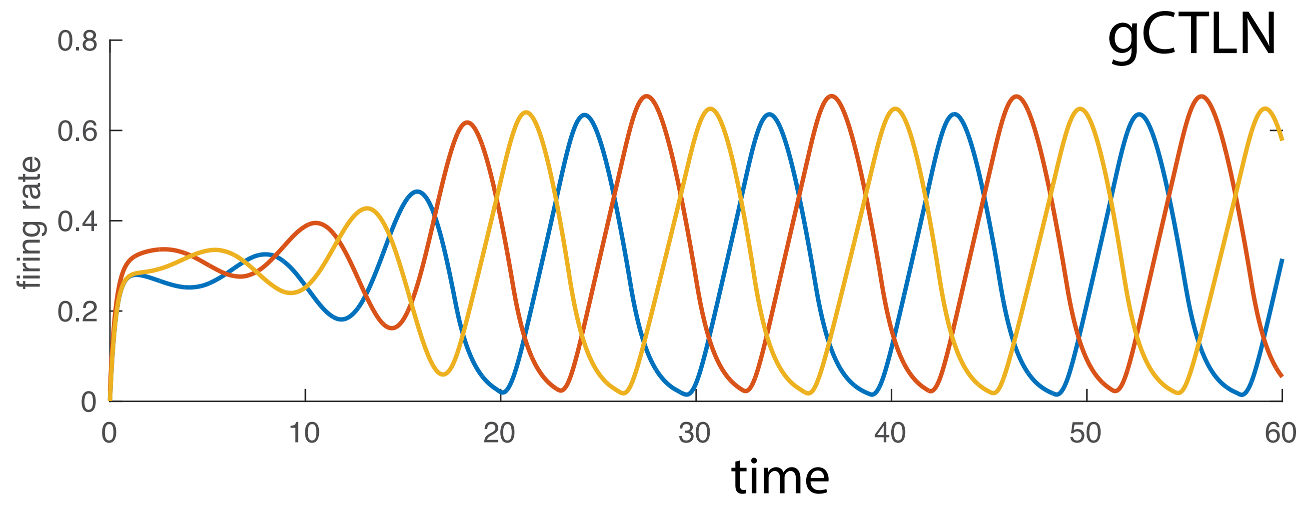

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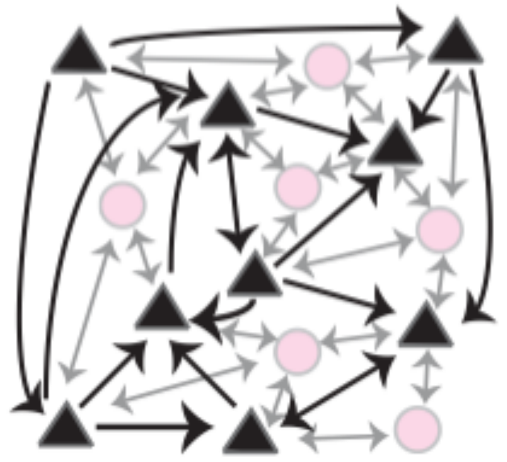

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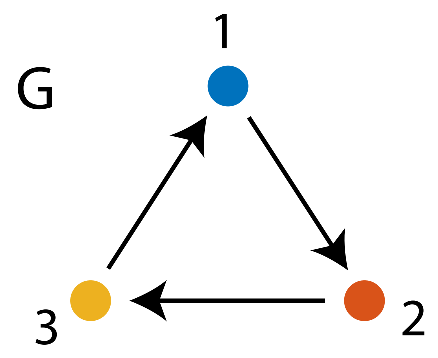

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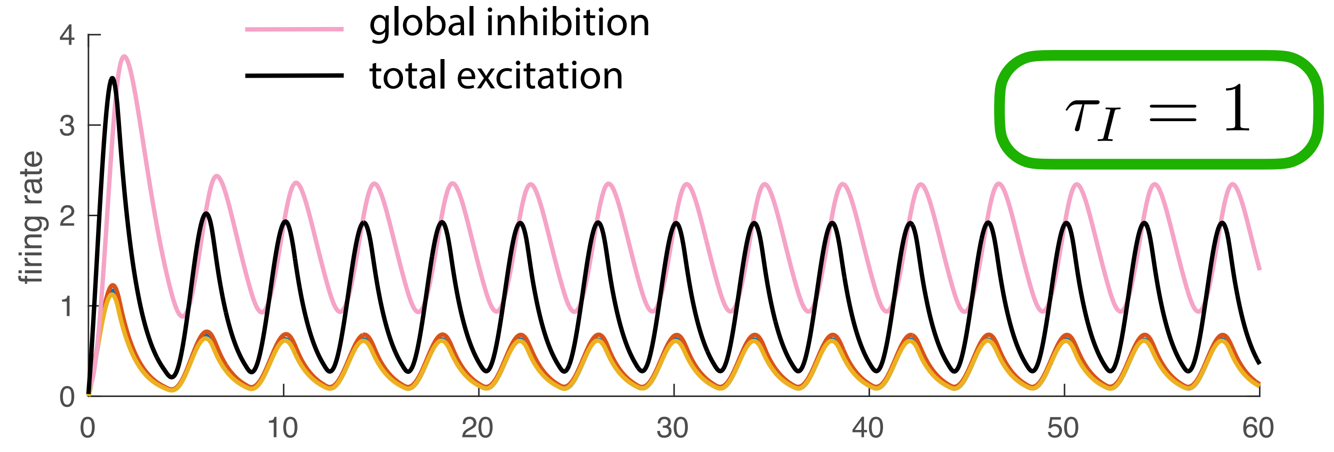

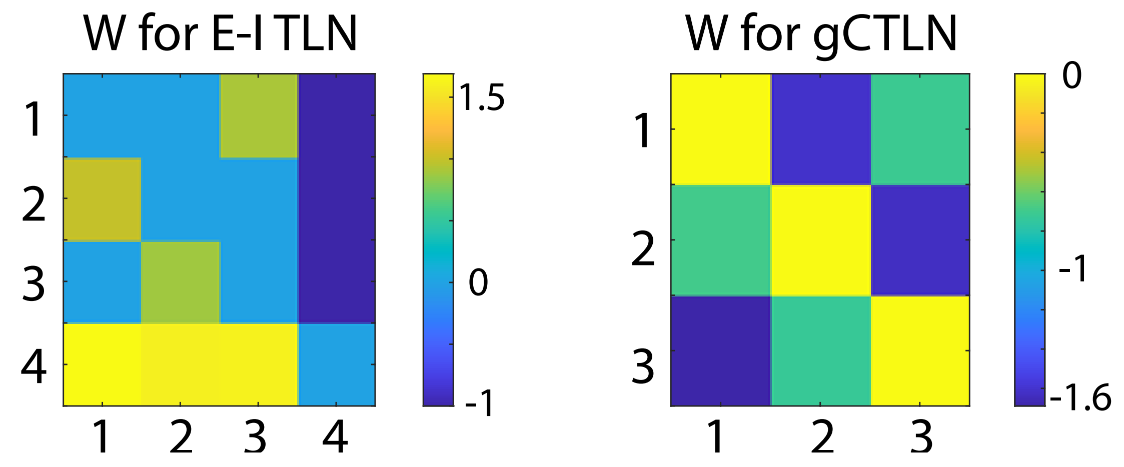

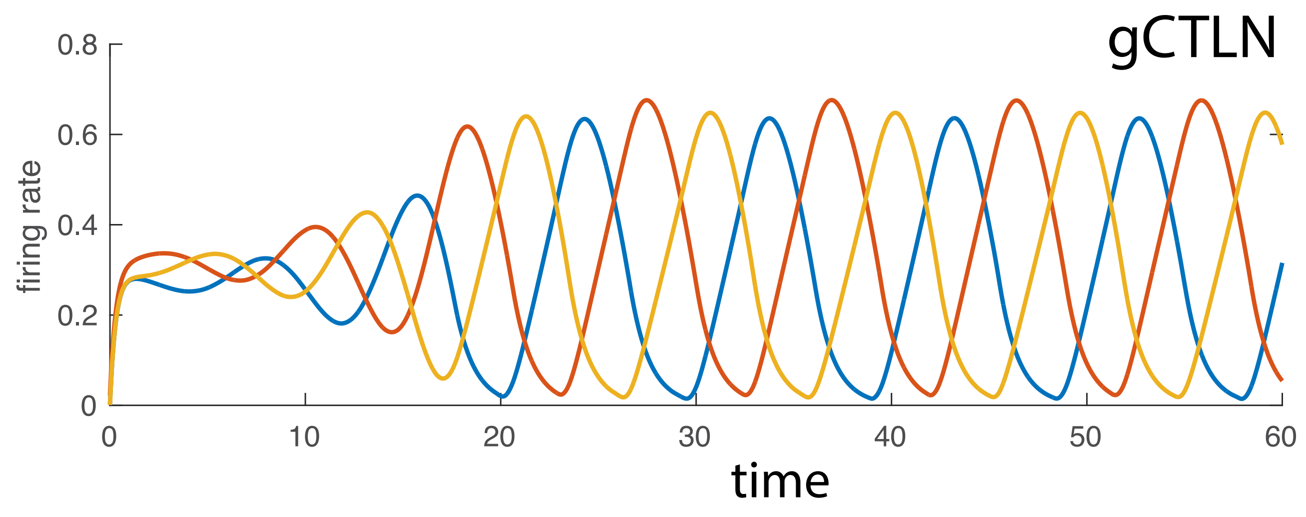

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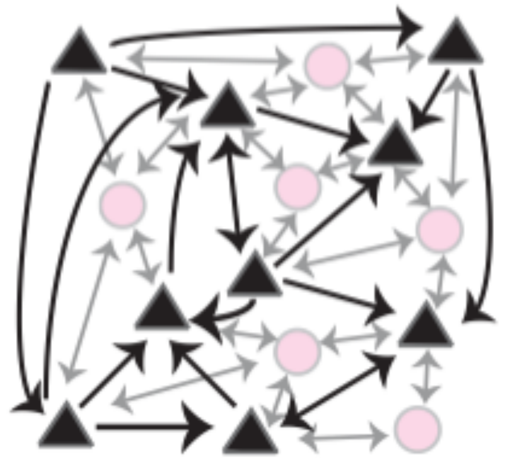

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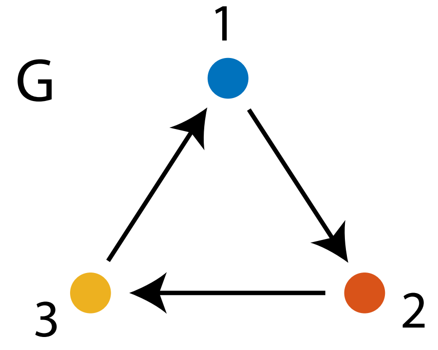

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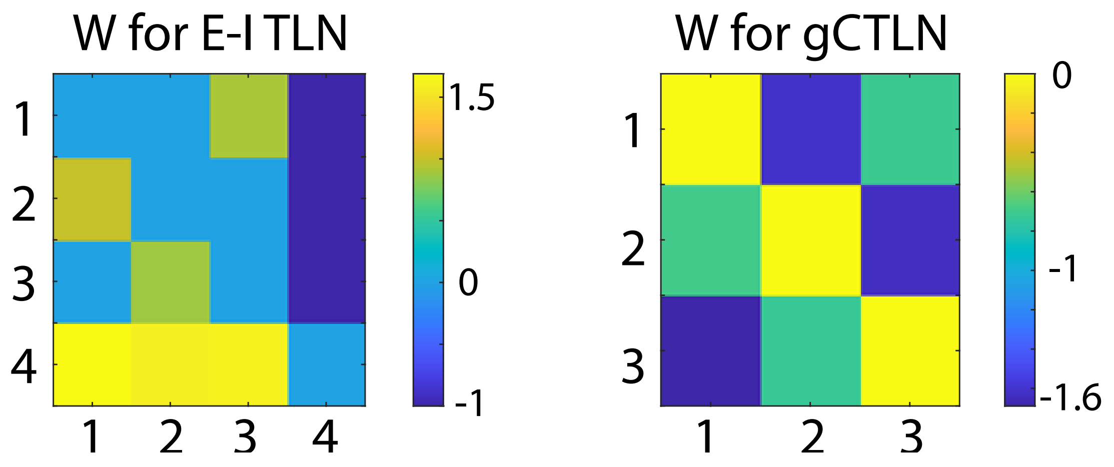

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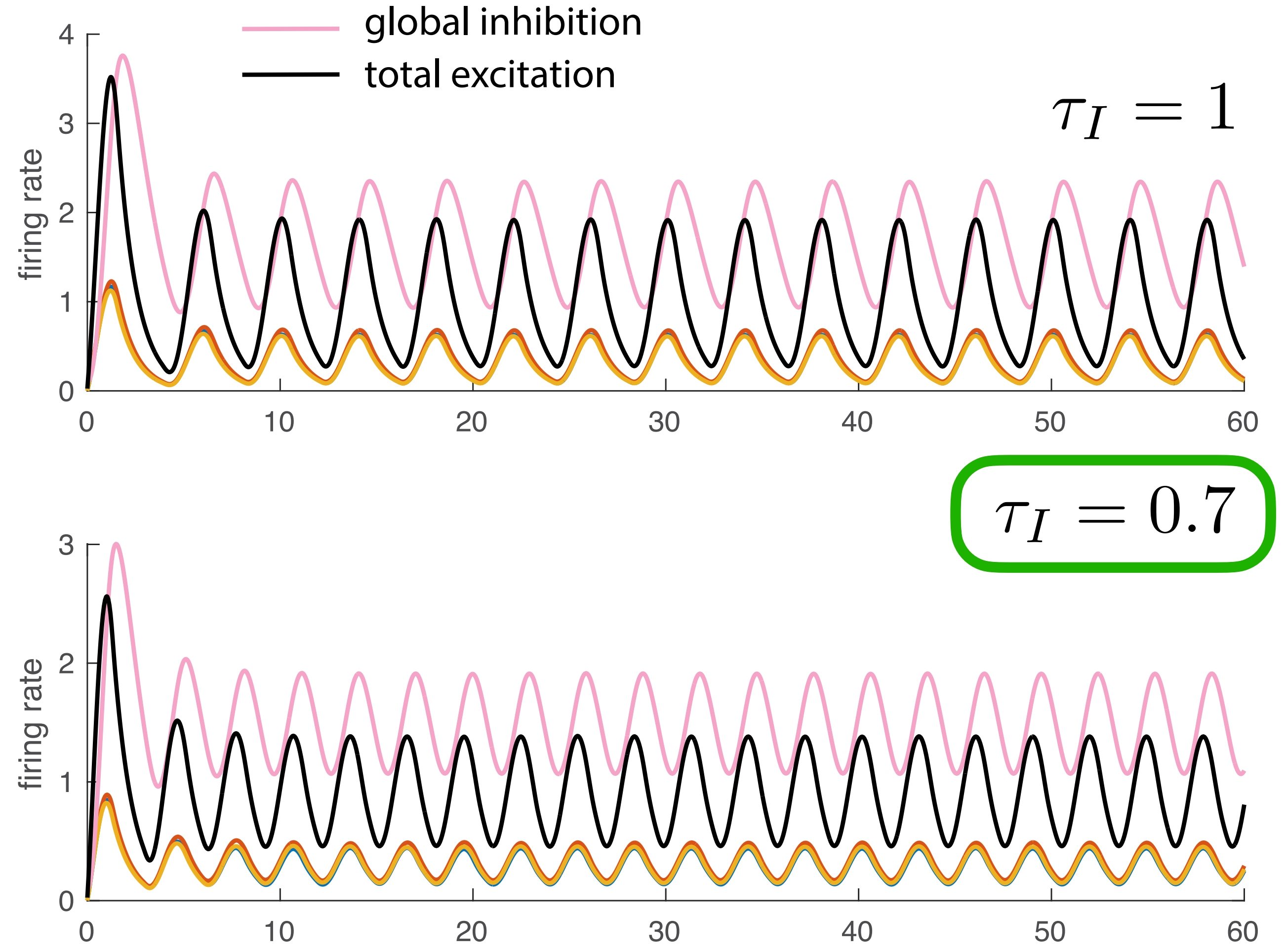

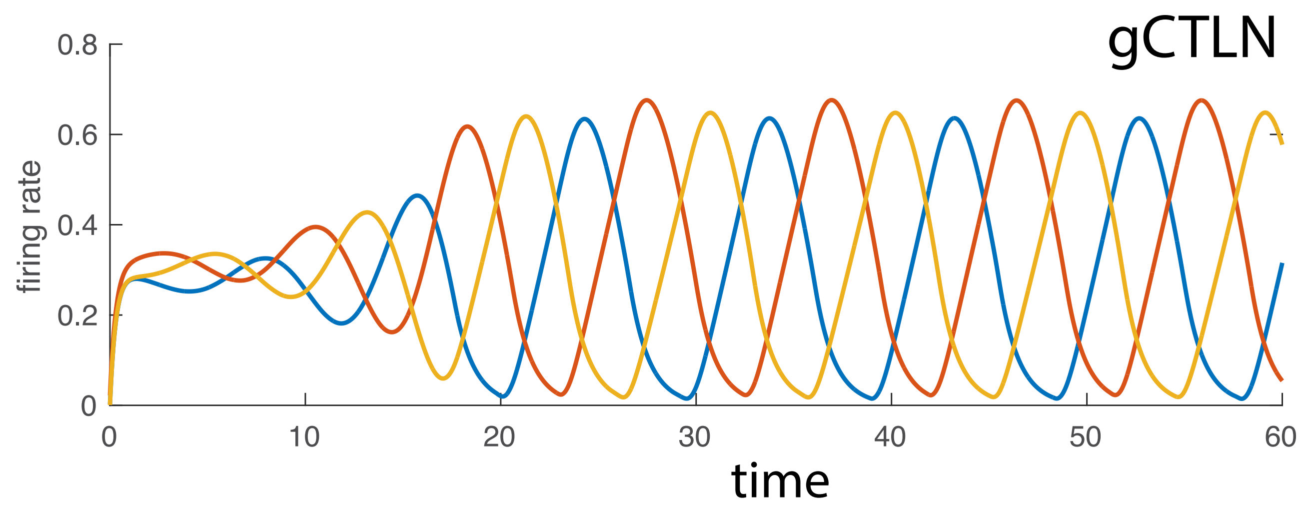

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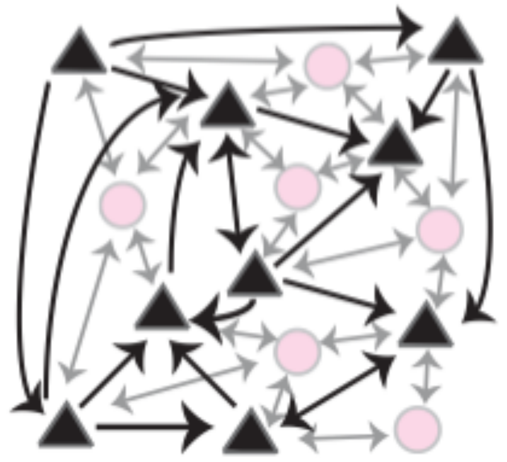

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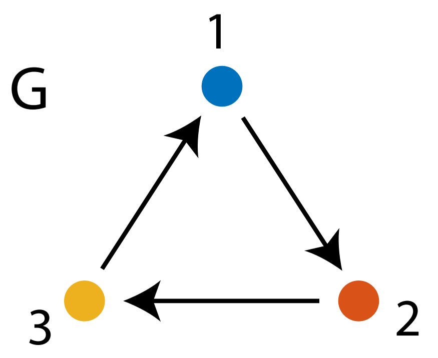
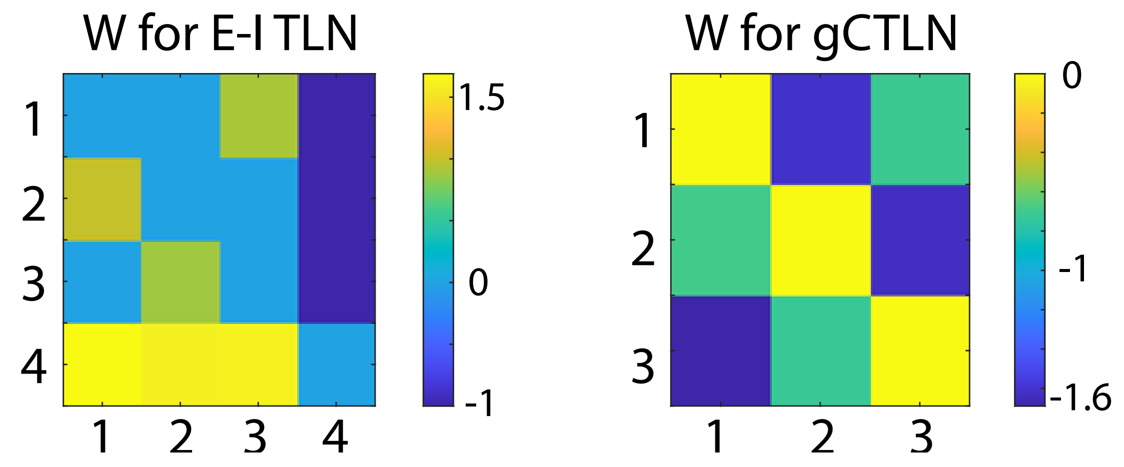


W for E-I TLN

$$W = \begin{pmatrix} 0 & 0 & a_3 & -1 \\ a_1 & 0 & 0 & -1 \\ 0 & a_2 & 0 & -1 \\ c_1 & c_2 & c_3 & 0 \end{pmatrix}$$



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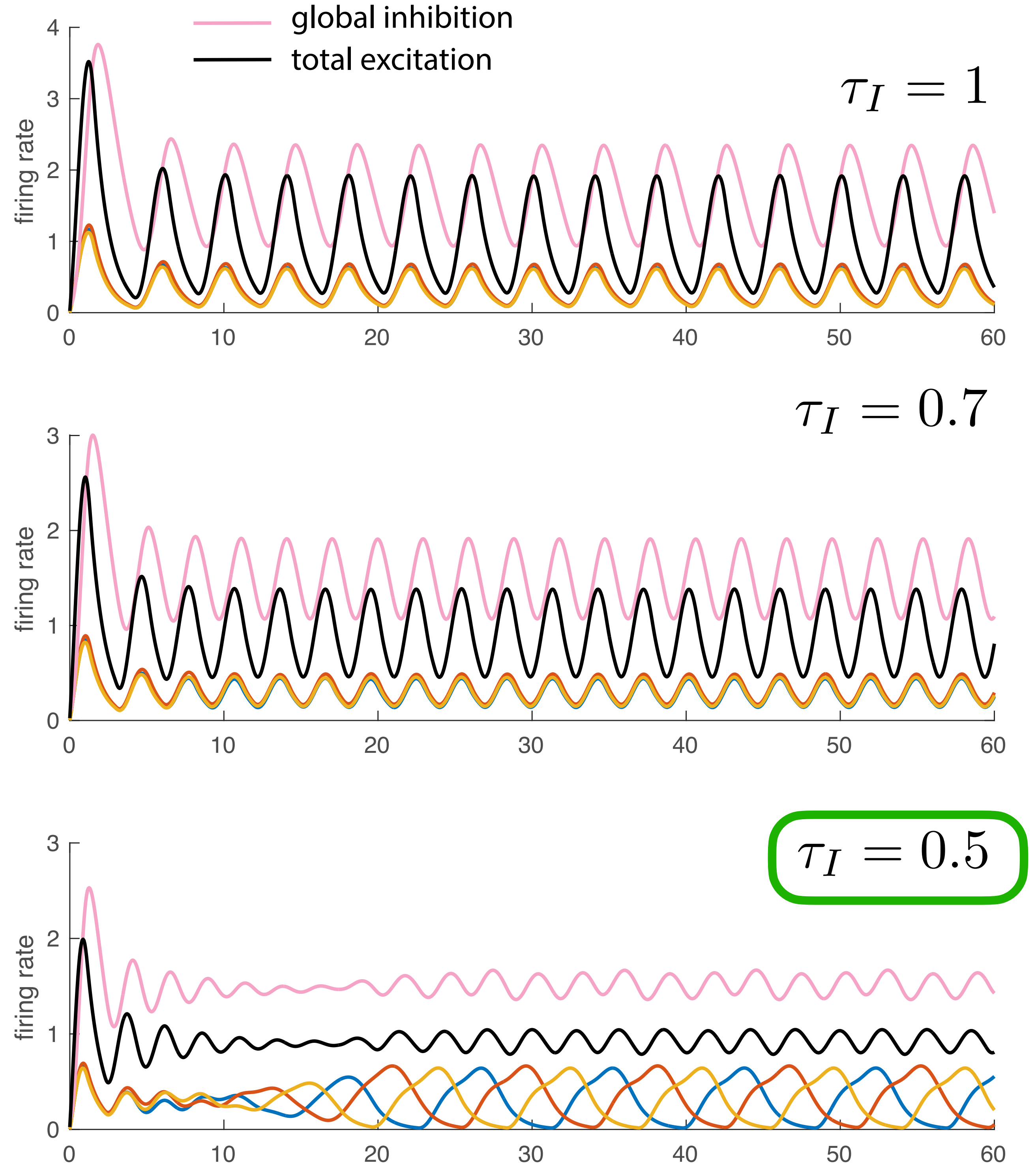
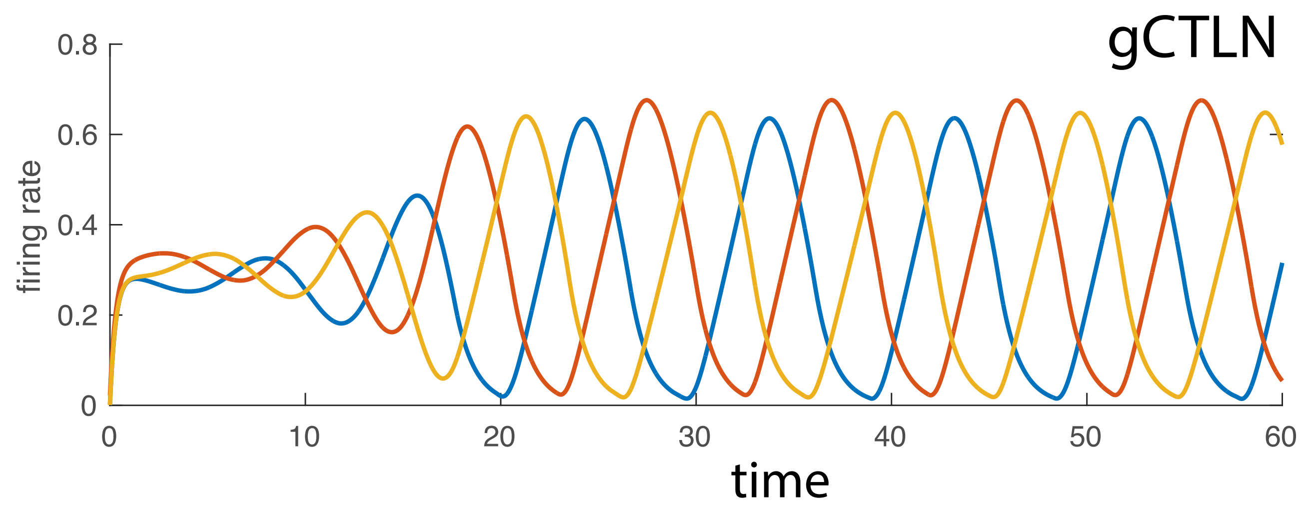
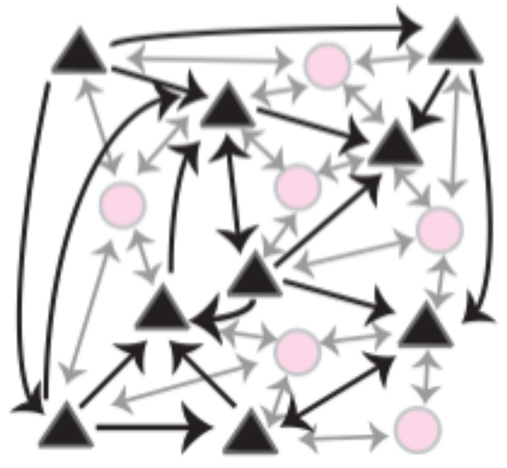




Beyond fixed points: do E-I TLNs produce similar dynamics to gCTLNs?

excitatory neurons
in a sea of inhibition



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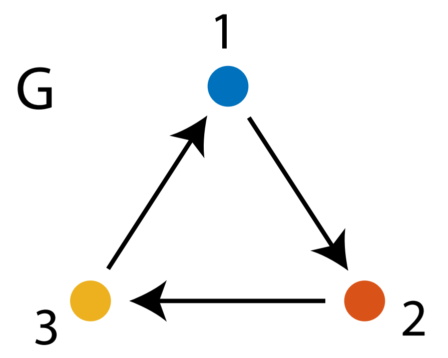

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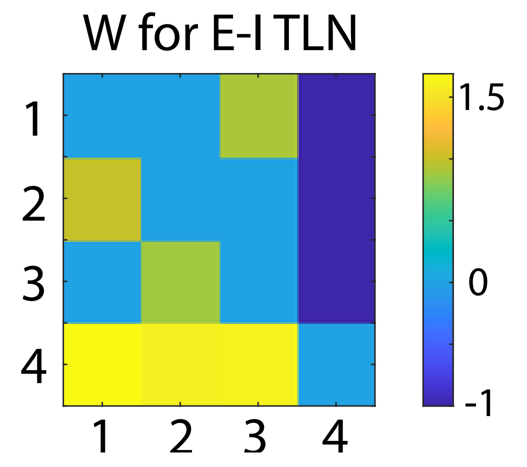
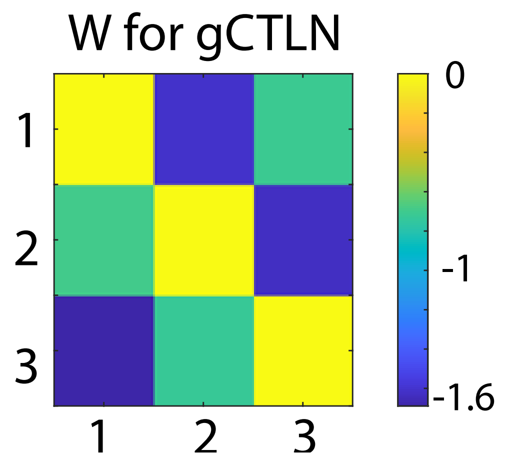
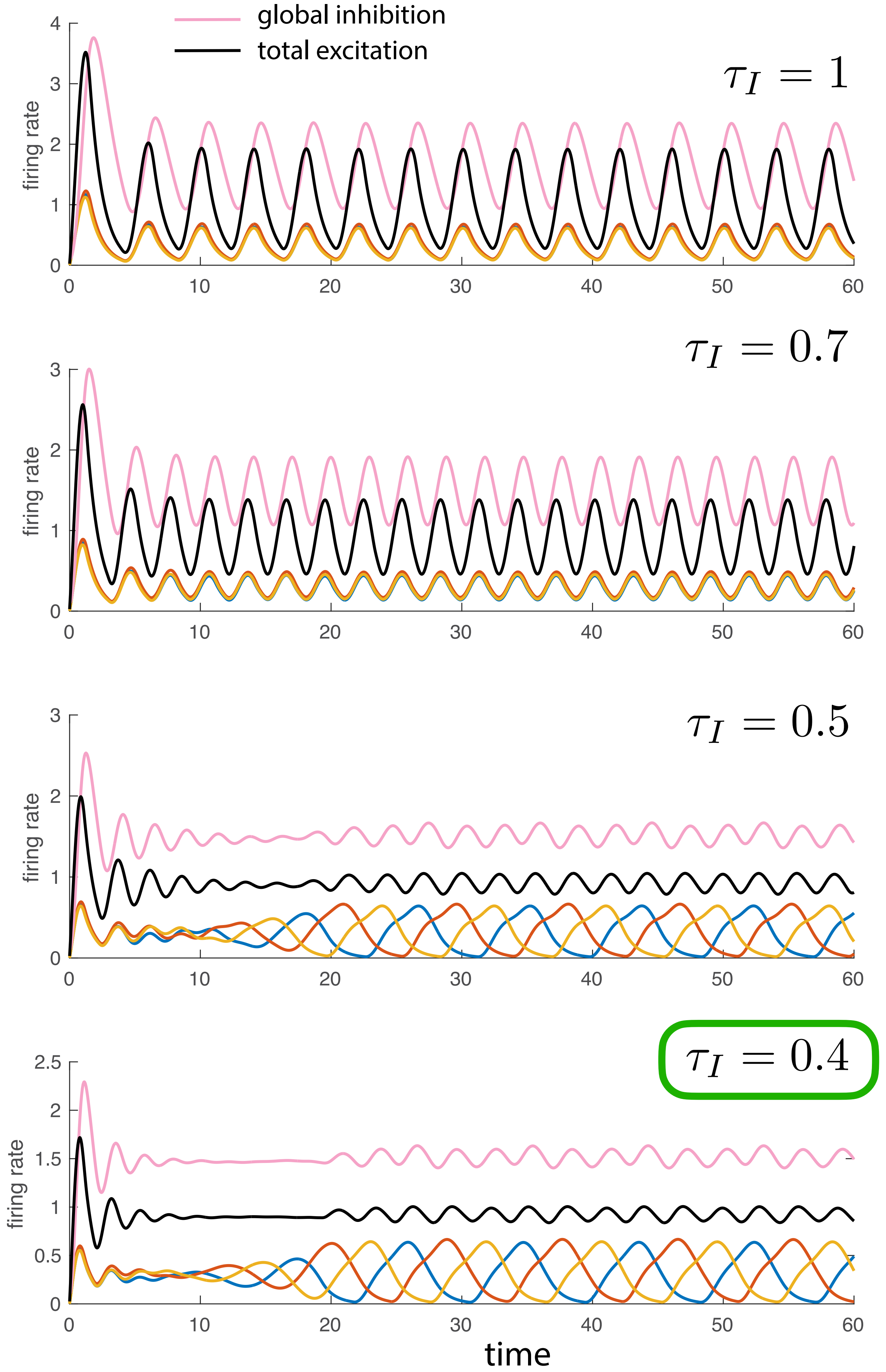
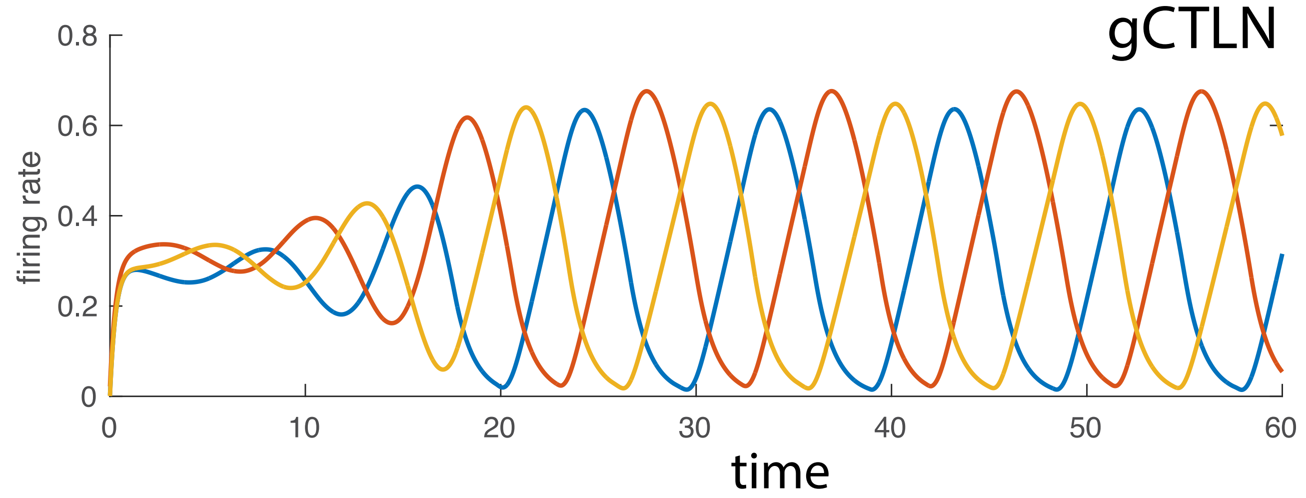




B





Beyond fixed points: do E-I TLNs produce similar dynamics to gCTLNs?

excitatory neurons
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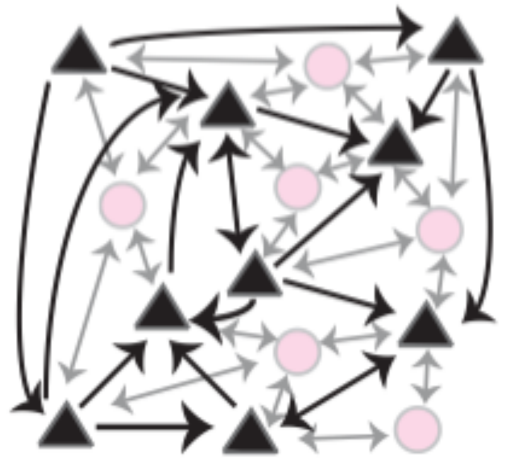

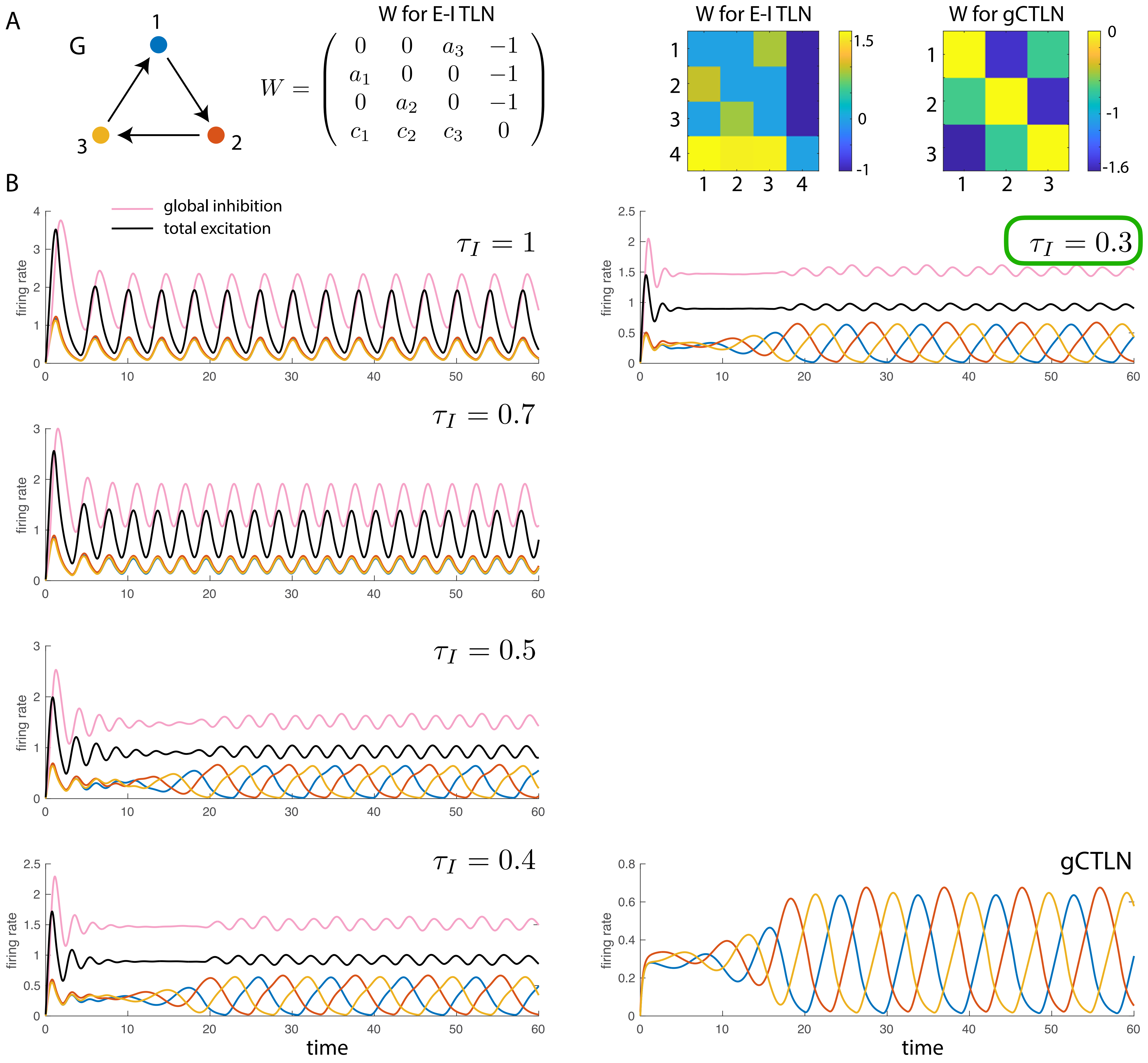

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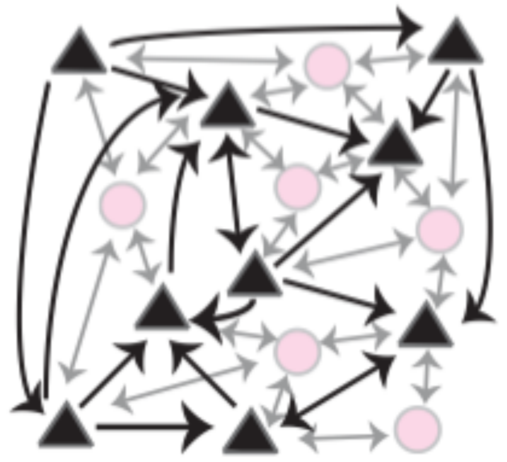

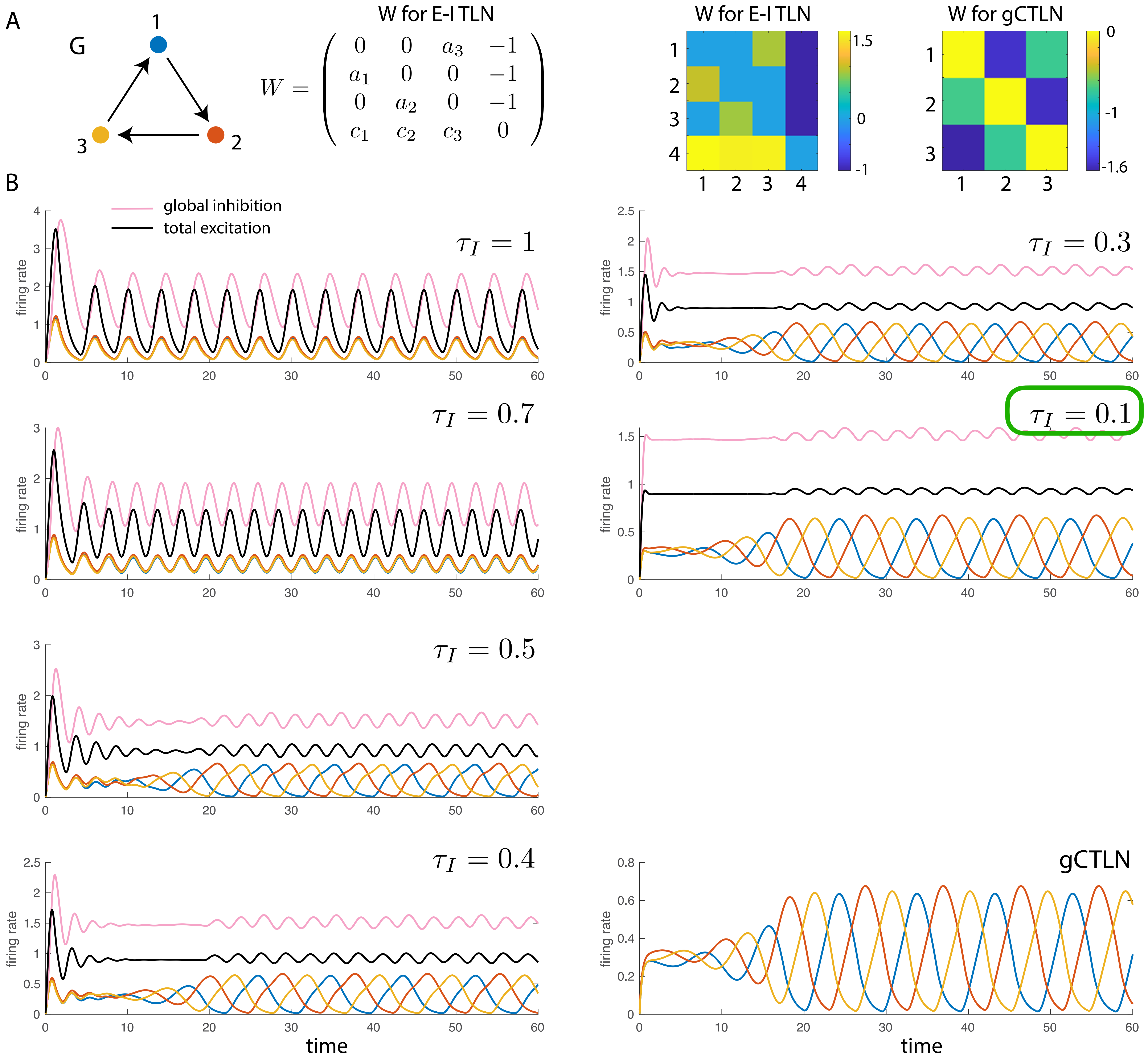

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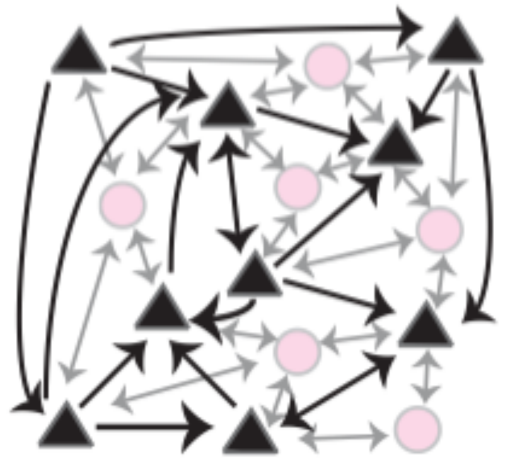
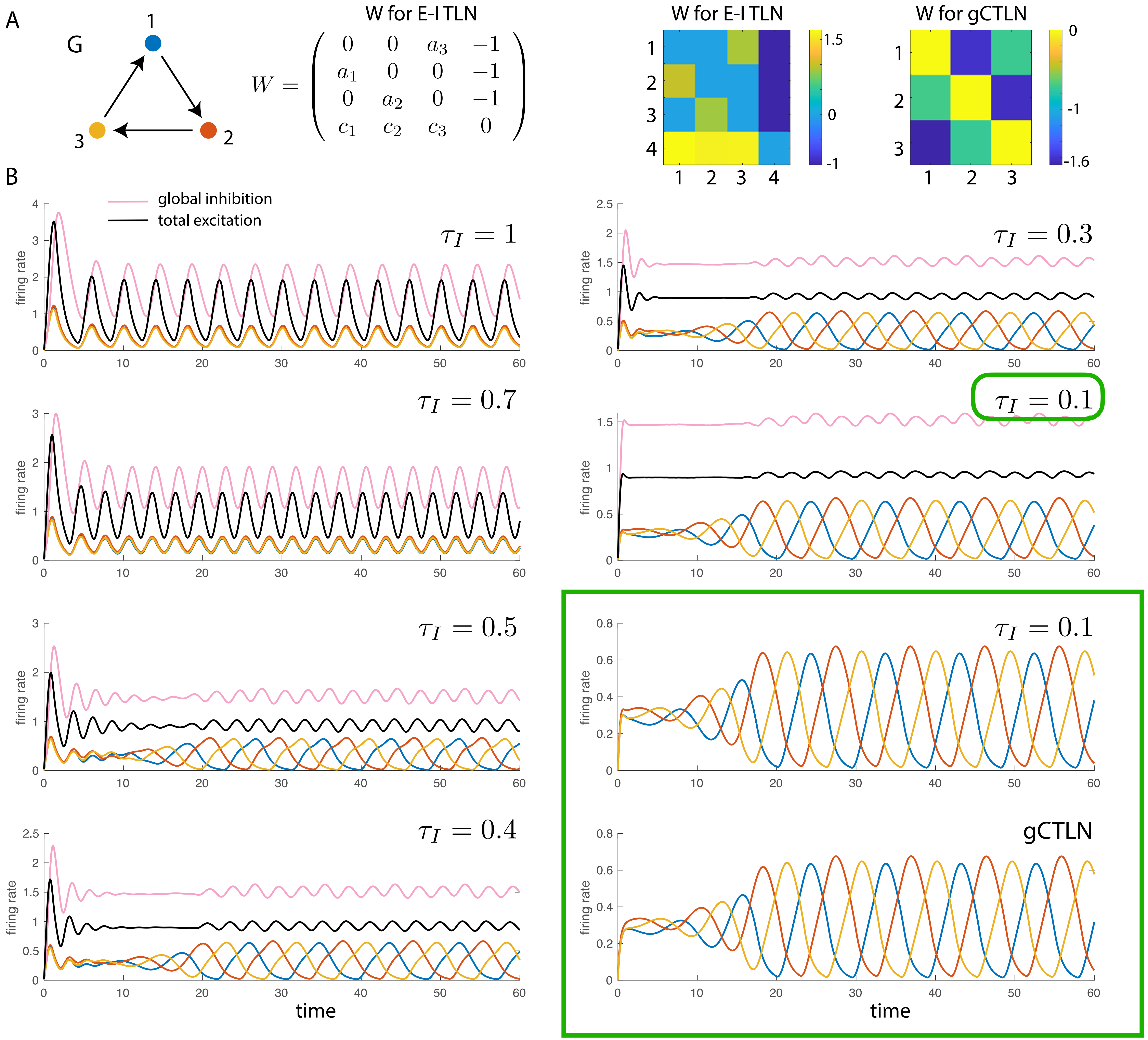
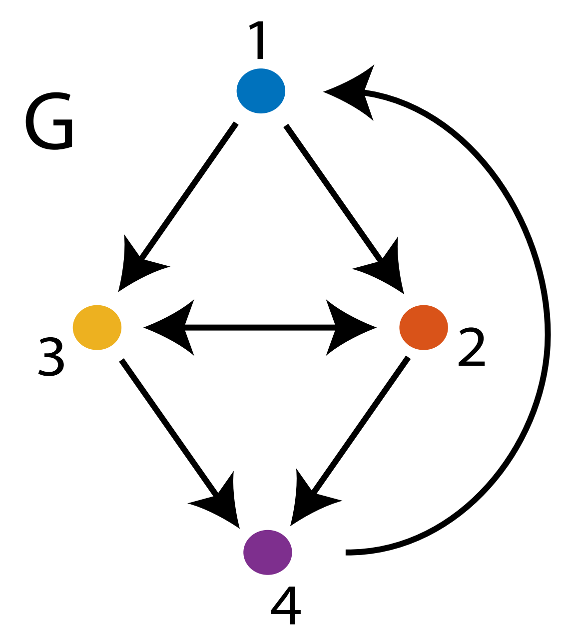




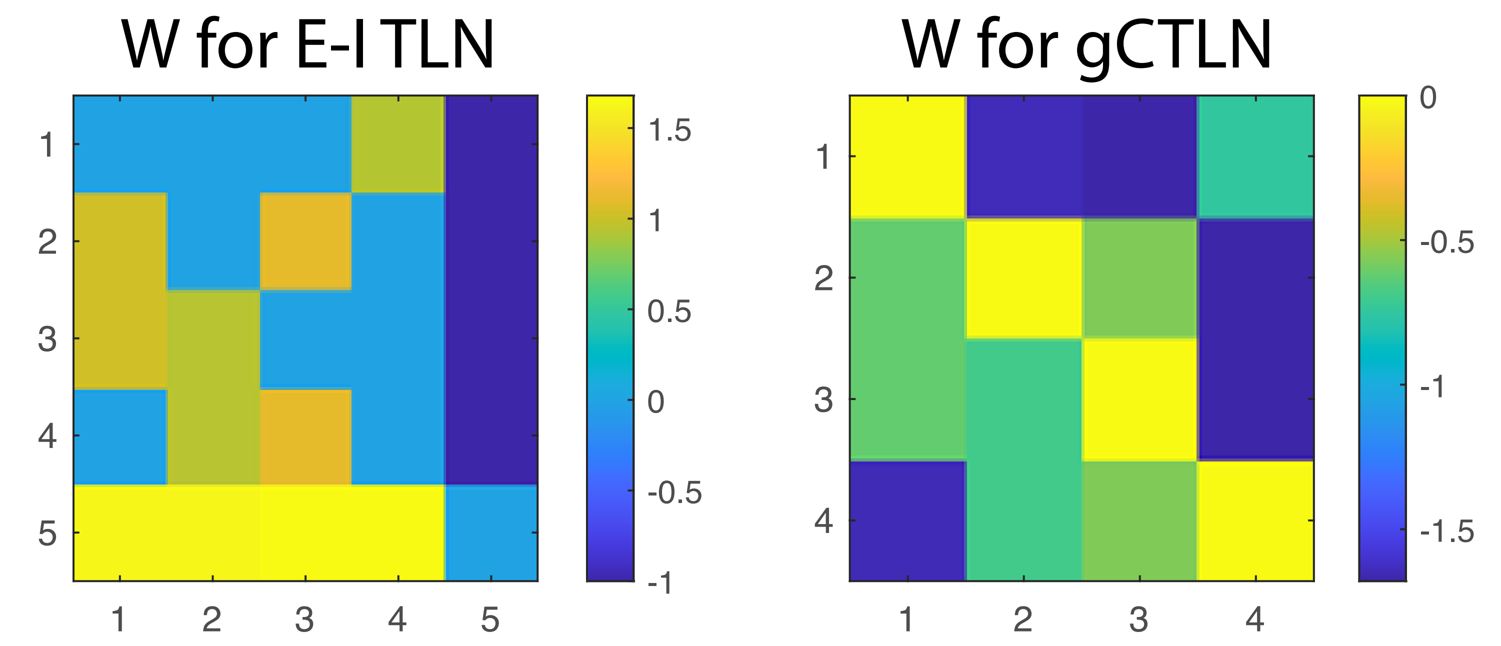
Beyond fixed points: do E-I TLNs produce similar dynamics to gCTLNs?

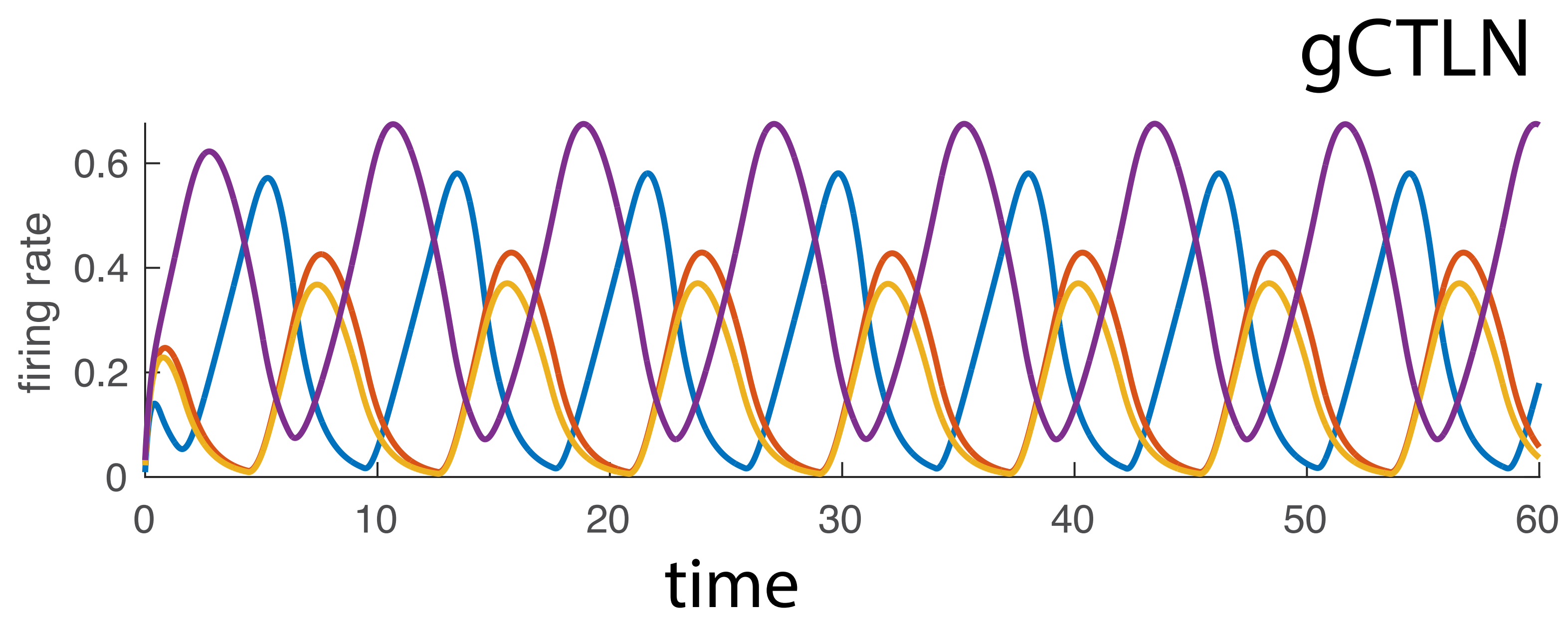
A



W for E-I TLN

$$W = \begin{pmatrix} 0 & 0 & 0 & a_4 & -1 \\ a_1 & 0 & a_3 & 0 & -1 \\ a_1 & a_2 & 0 & 0 & -1 \\ 0 & a_2 & a_3 & 0 & -1 \\ c_1 & c_2 & c_3 & c_4 & 0 \end{pmatrix}$$





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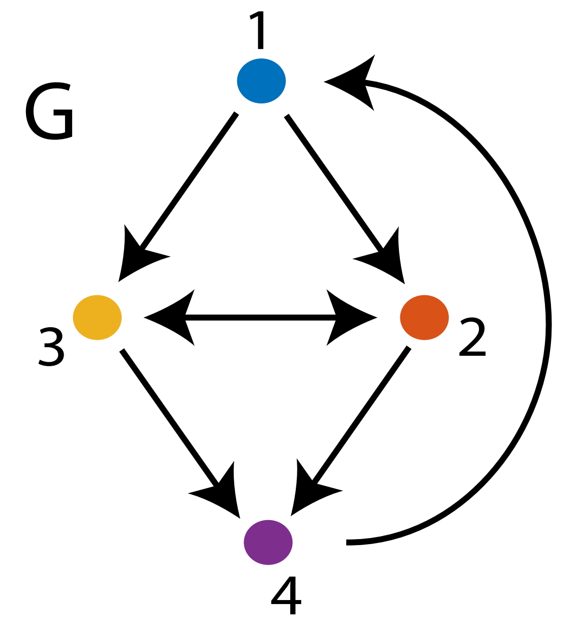

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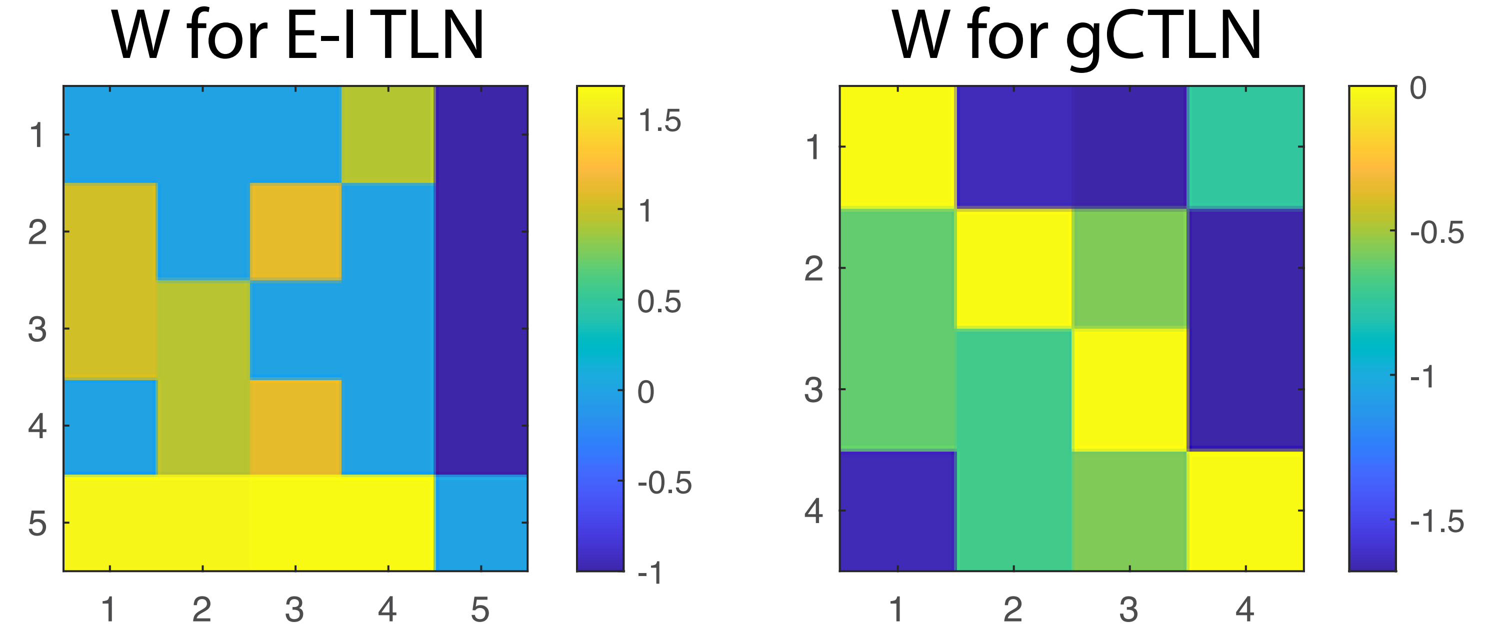

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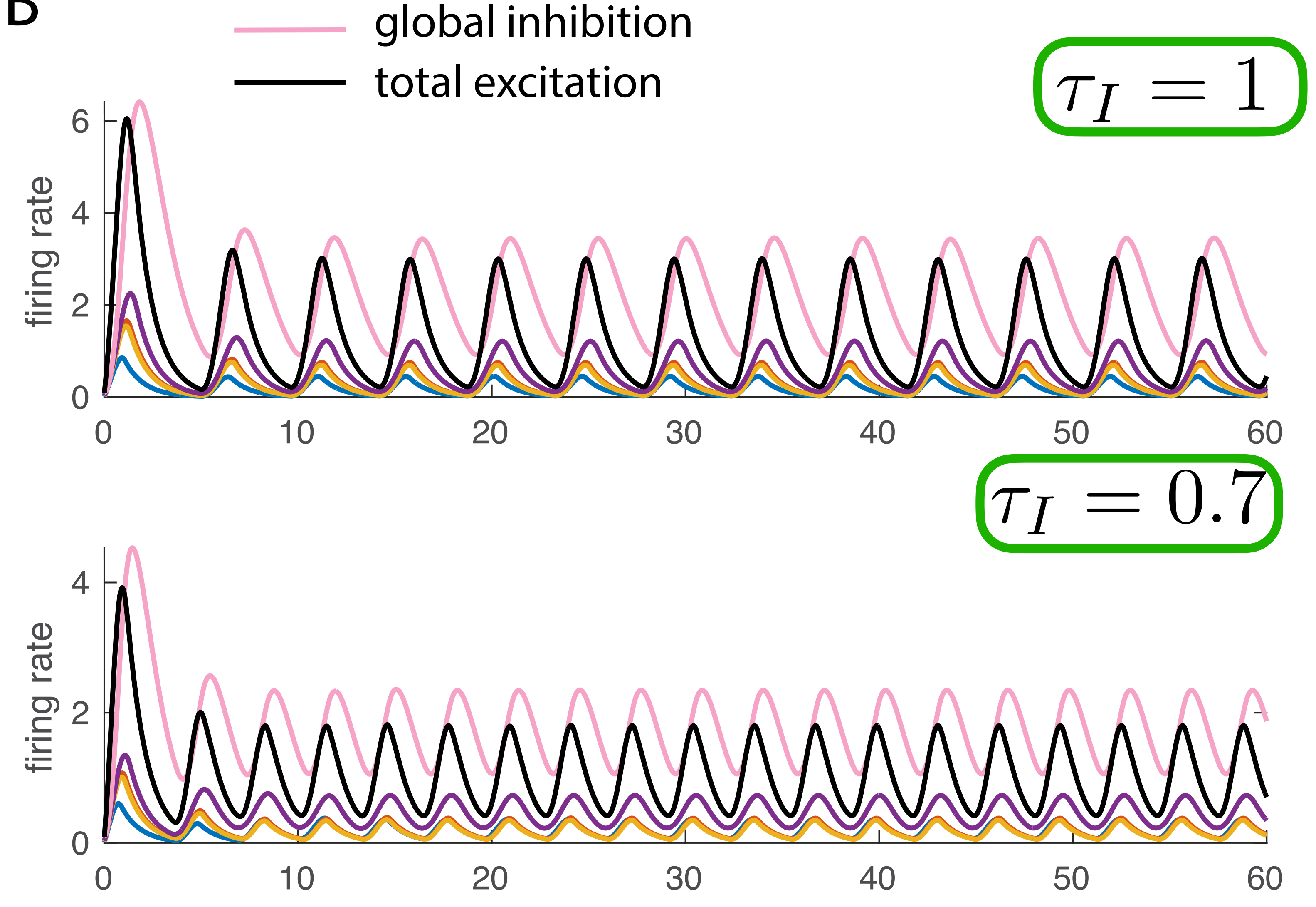

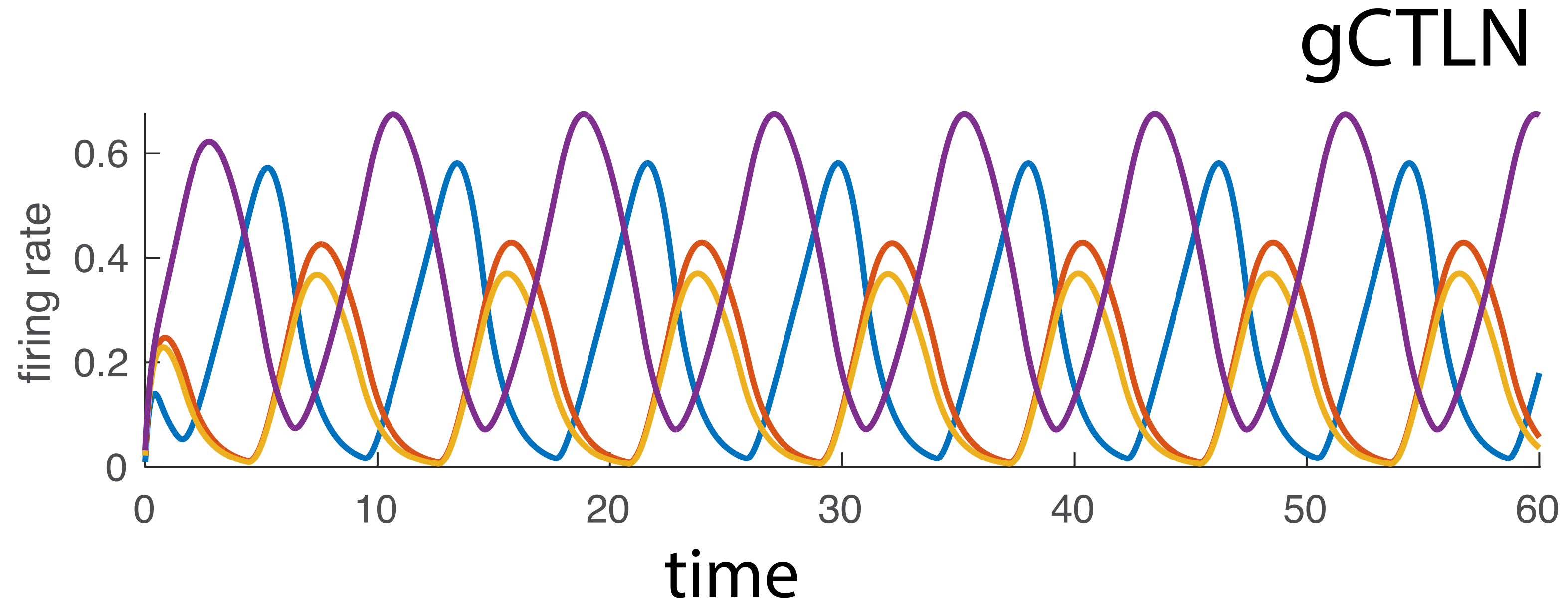

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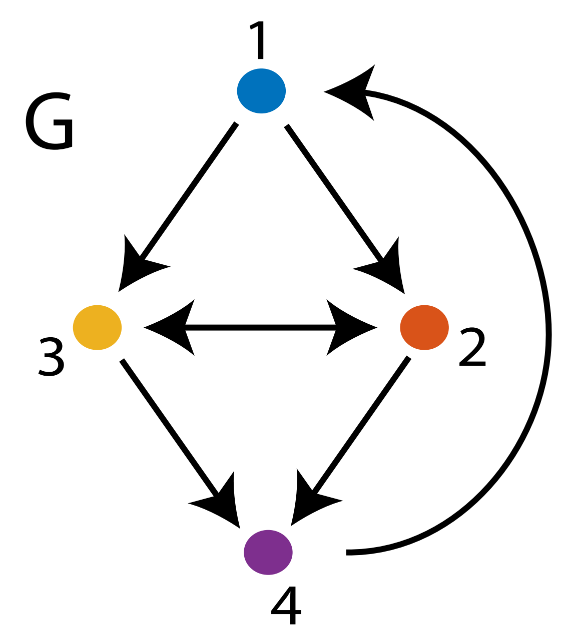
A



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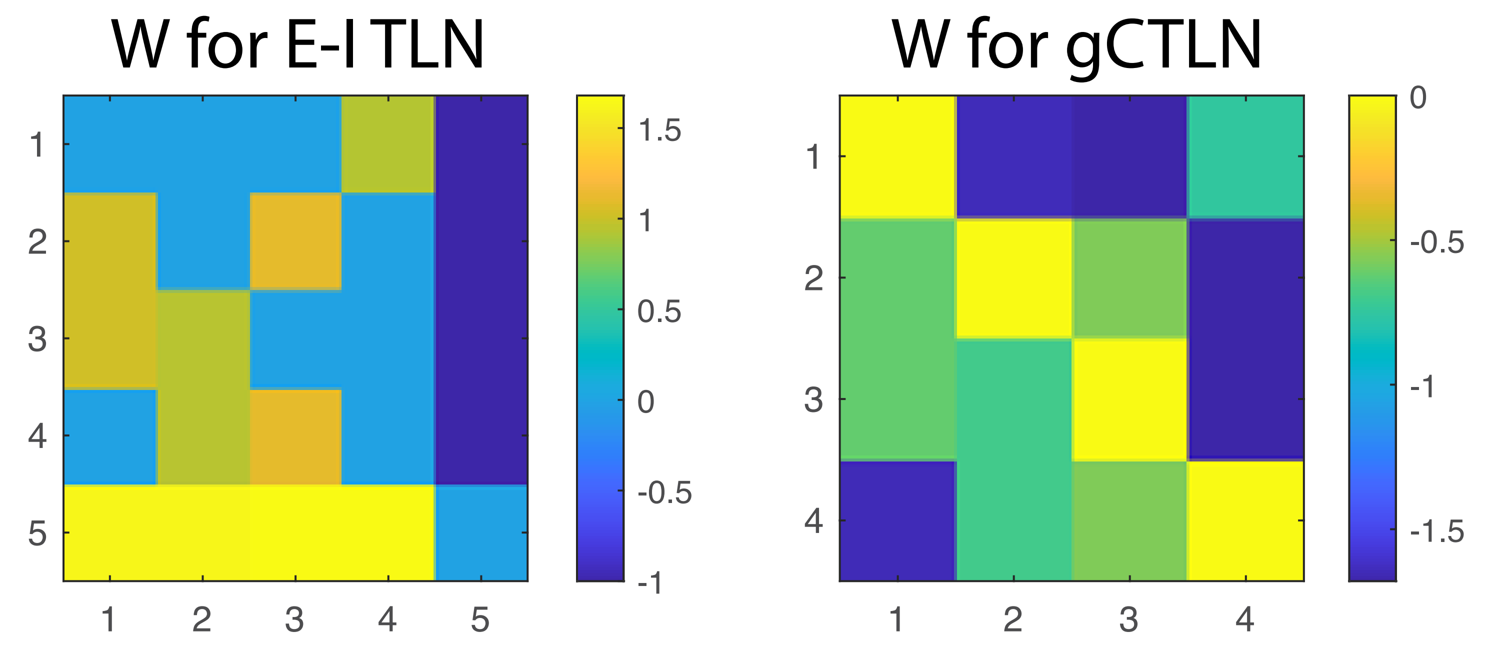

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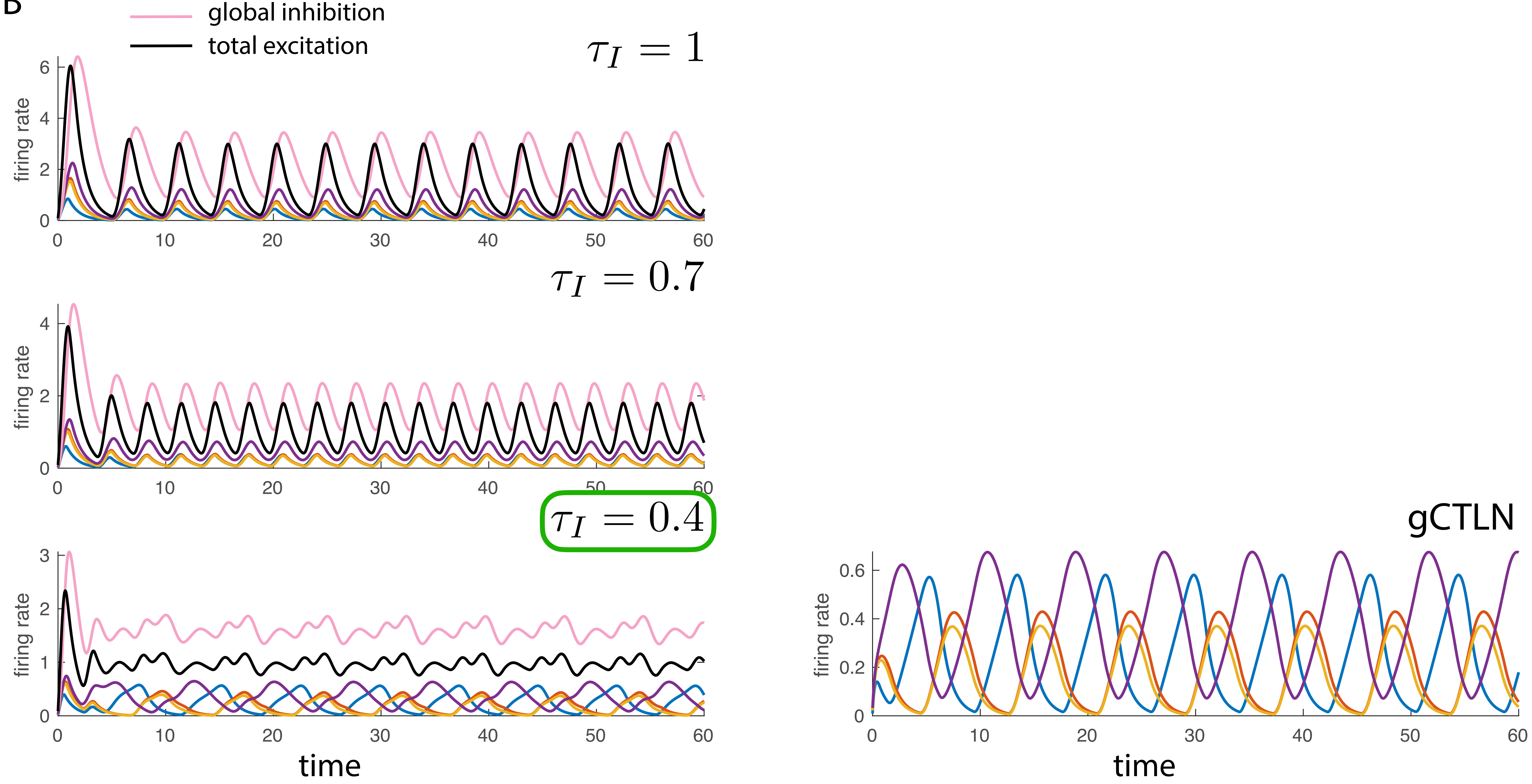

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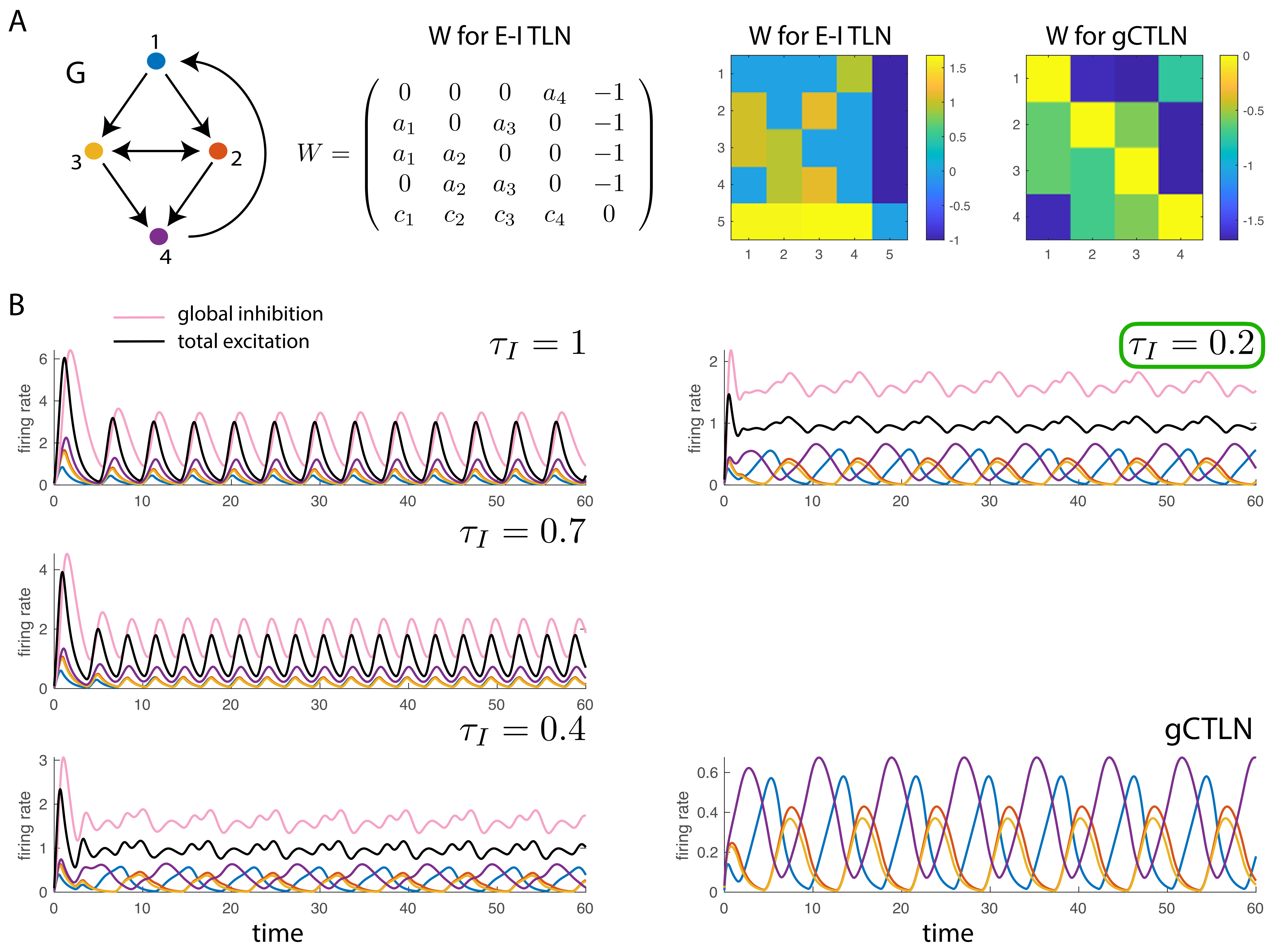

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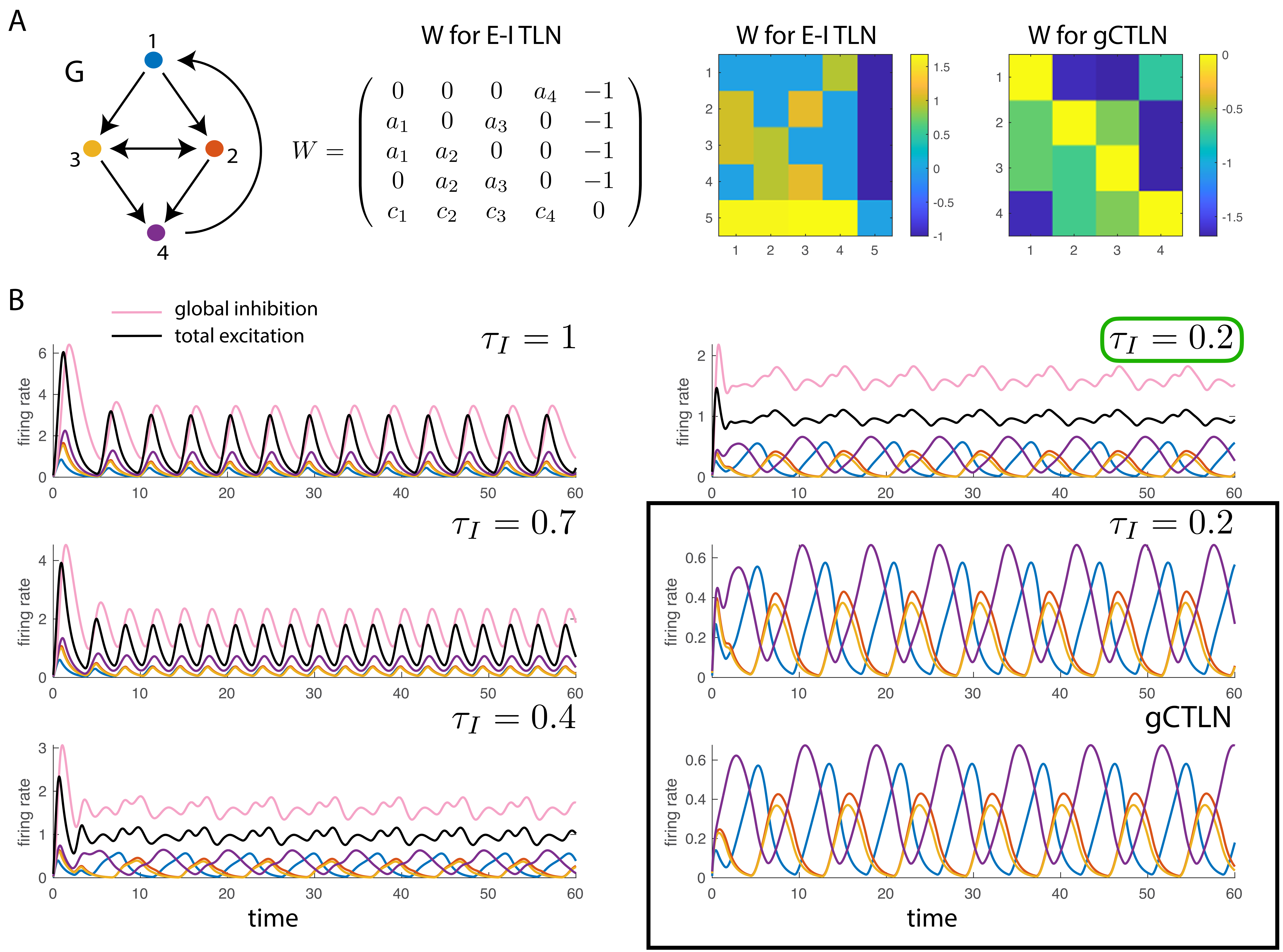
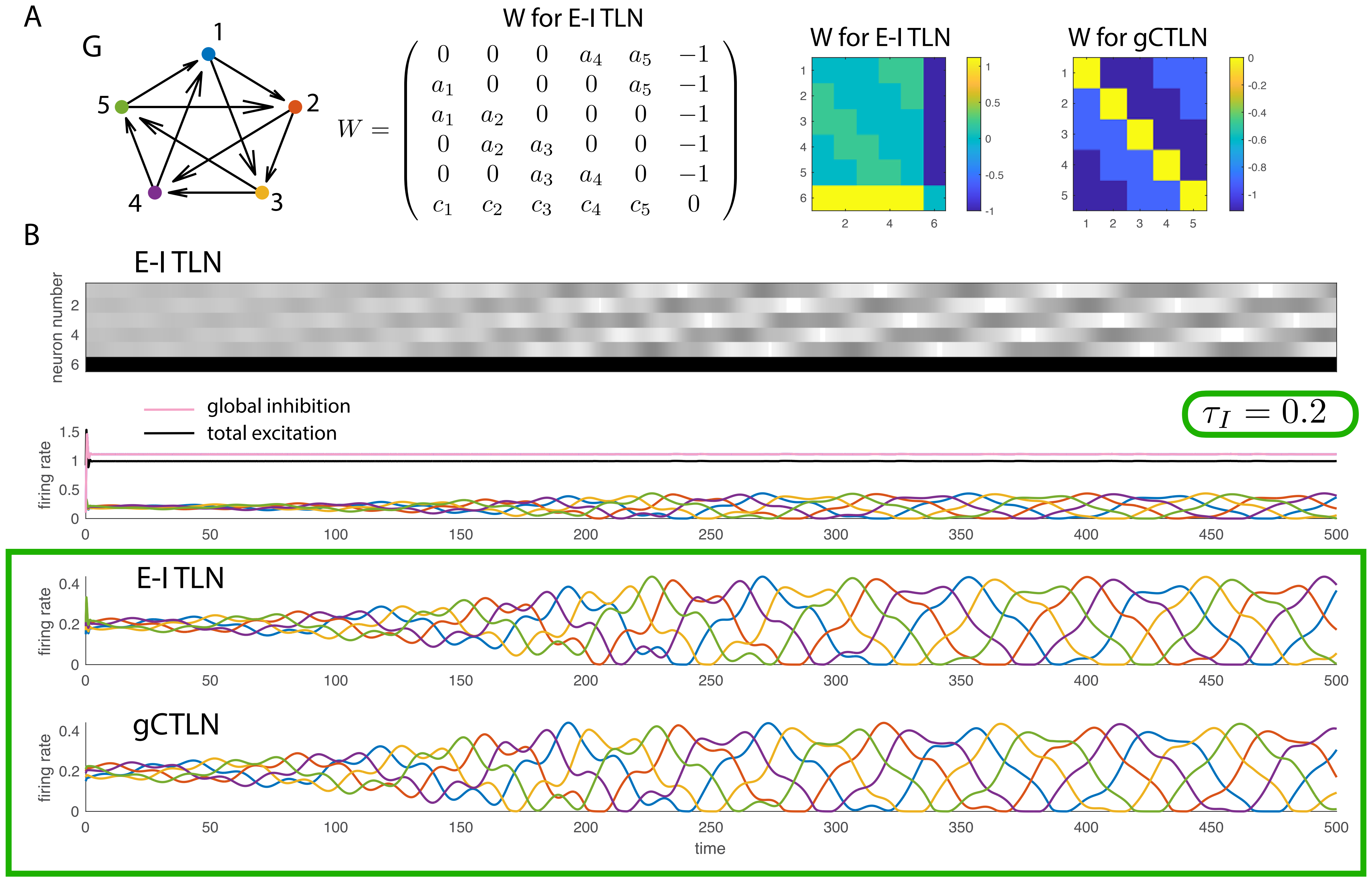
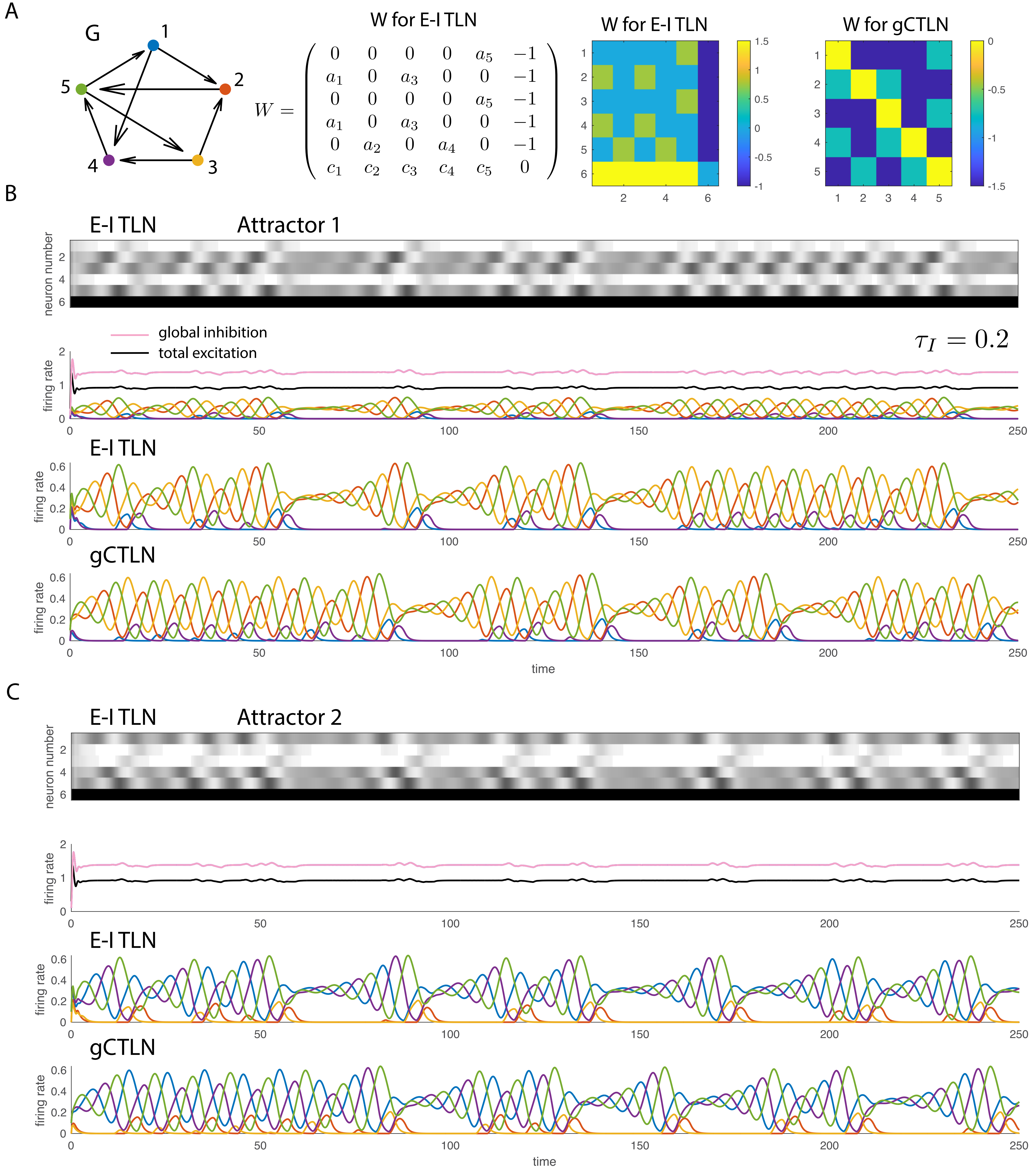


Even "exotic" attractors like Gaudi and baby chaos look the same





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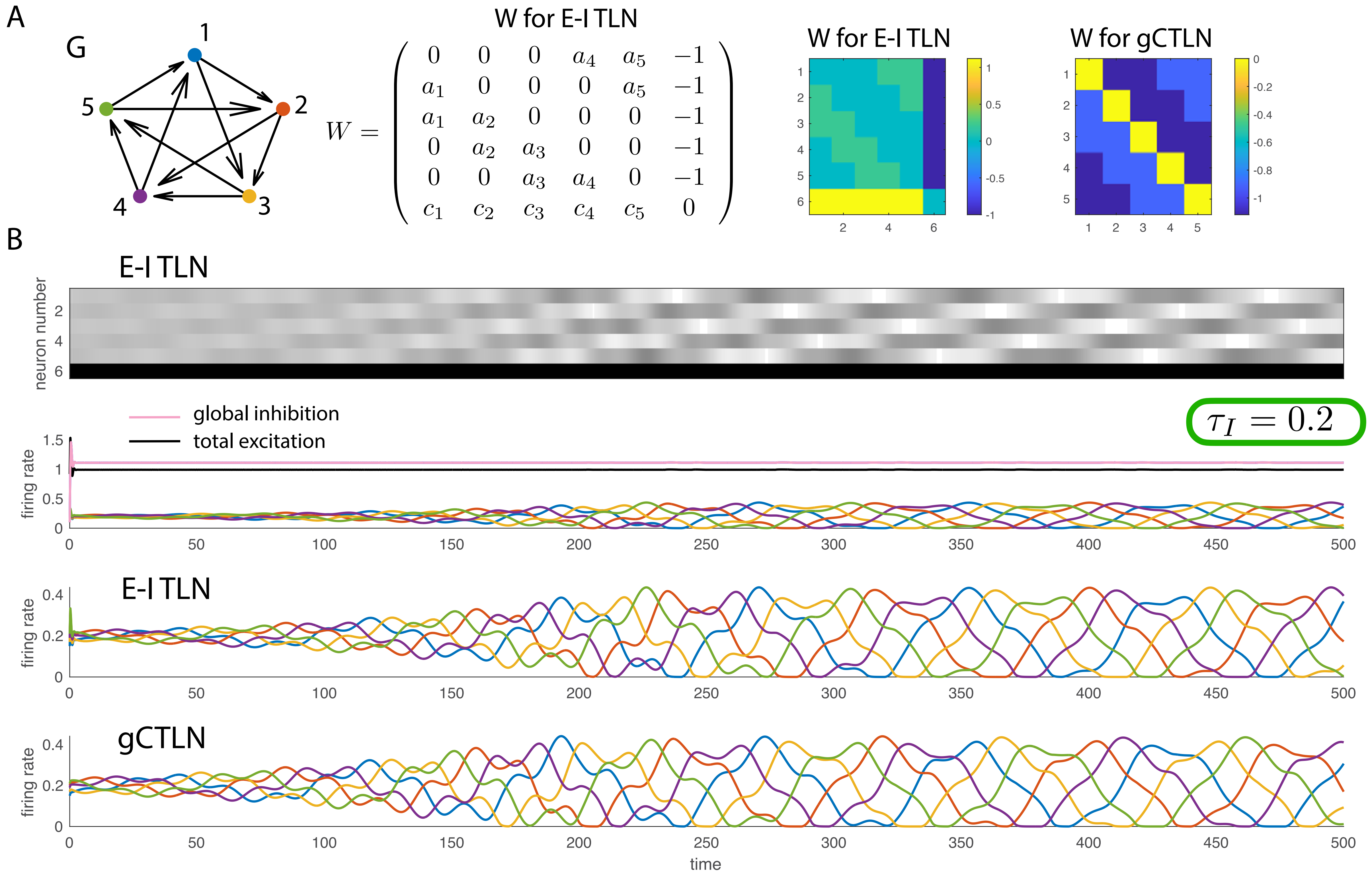

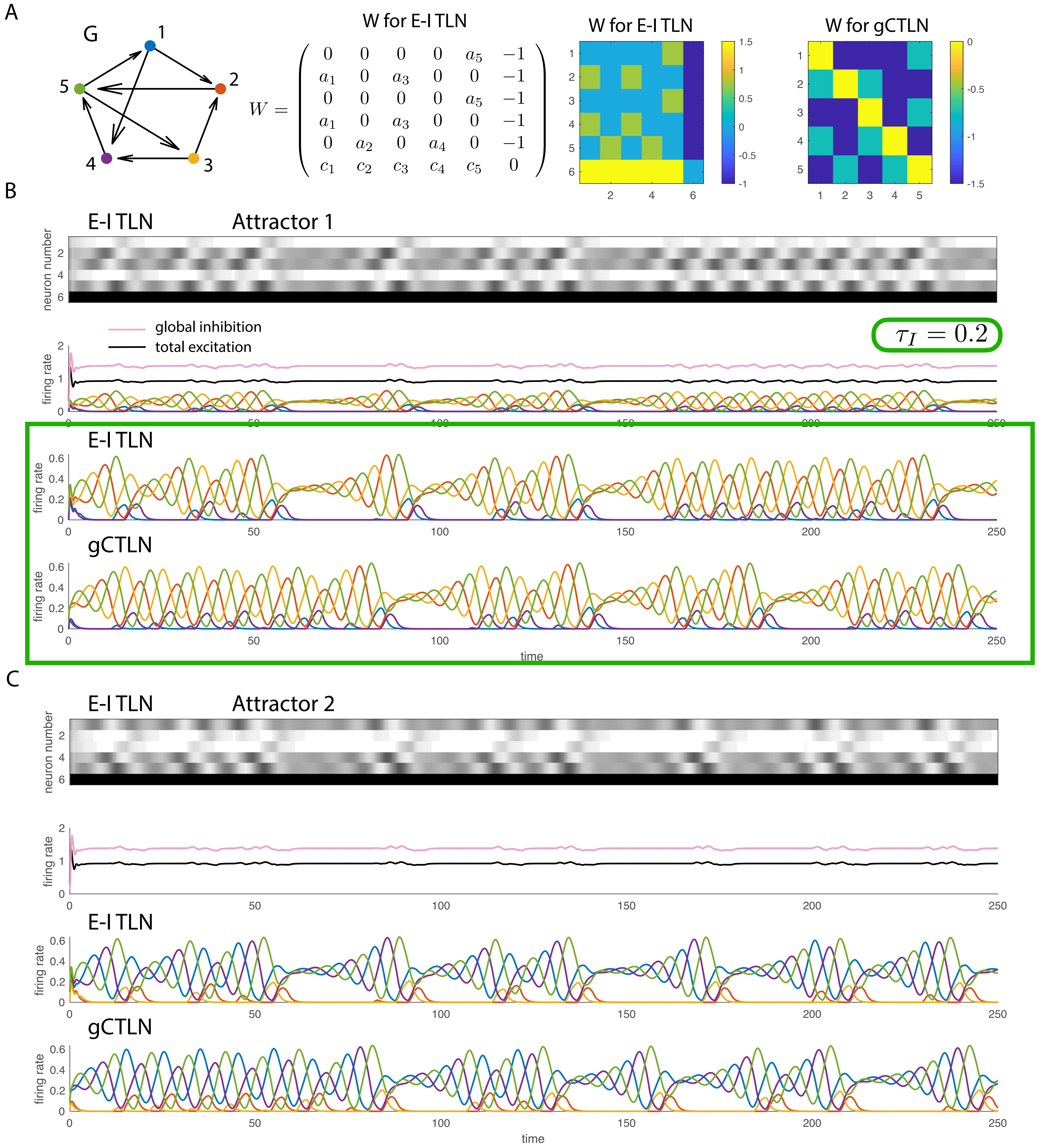

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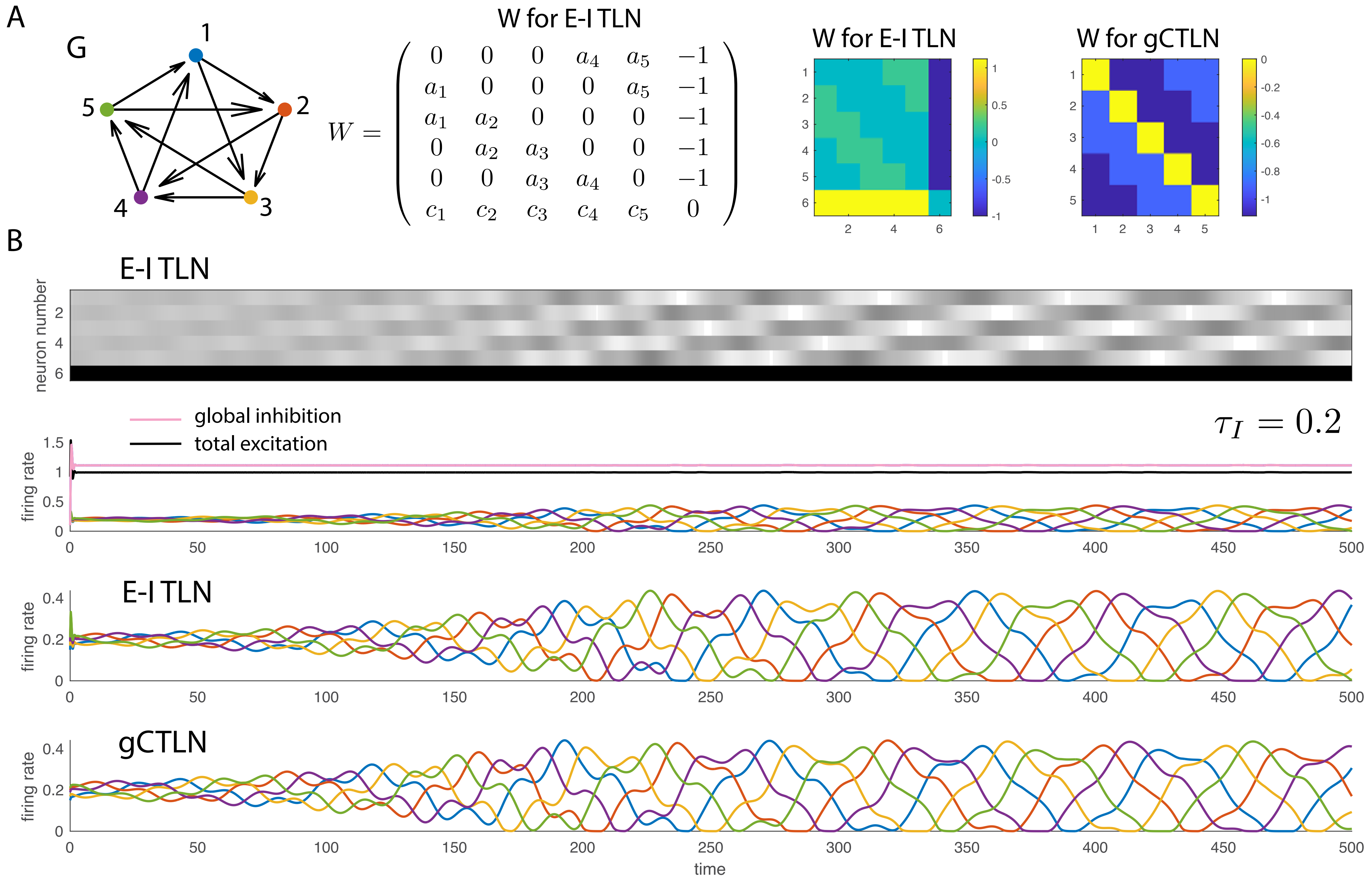
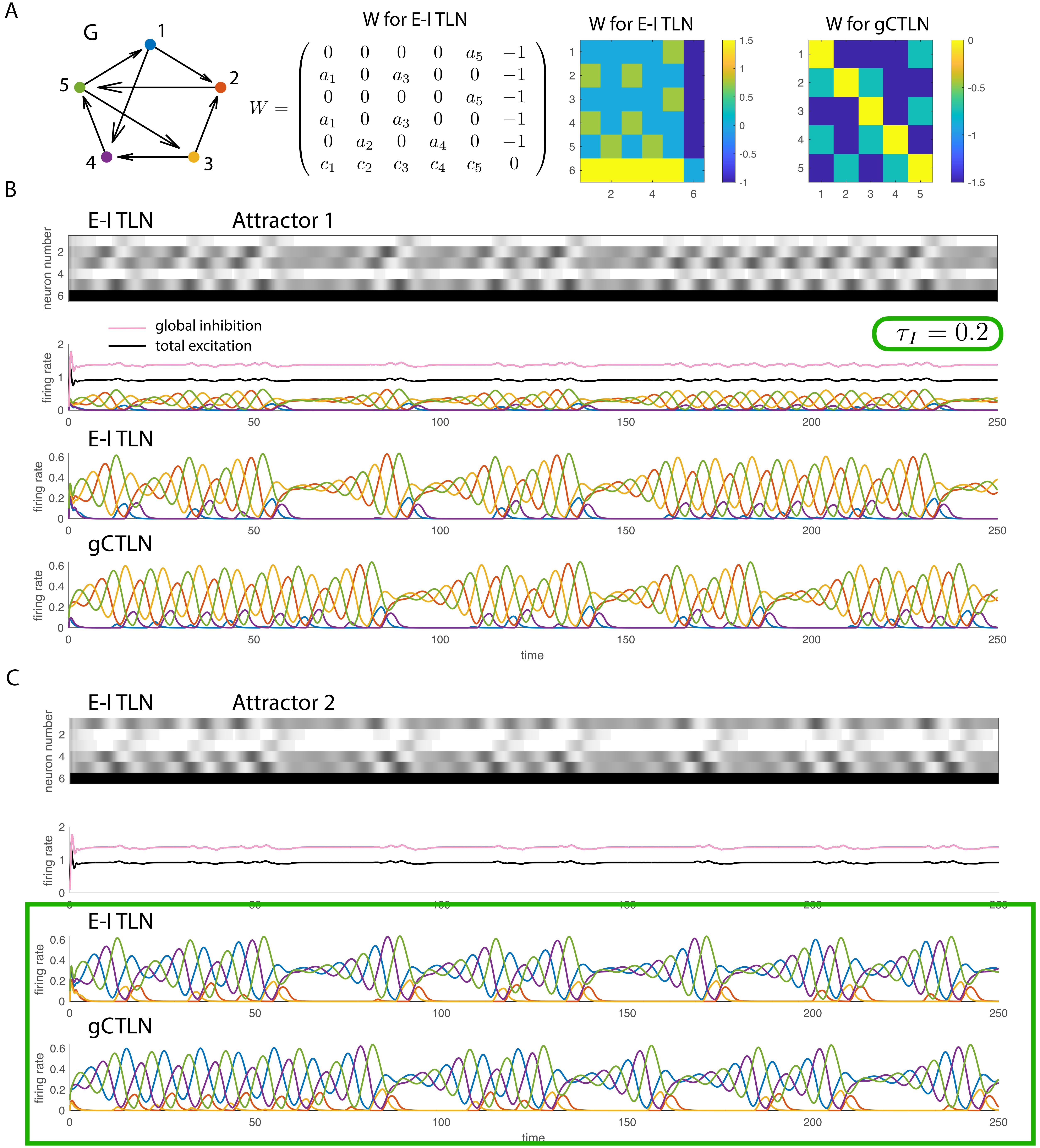
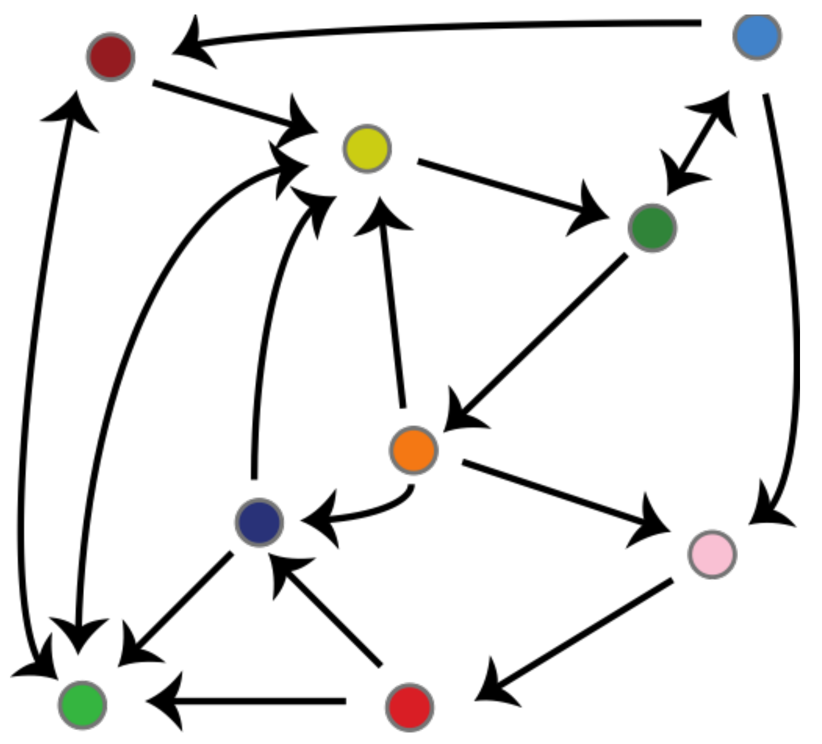
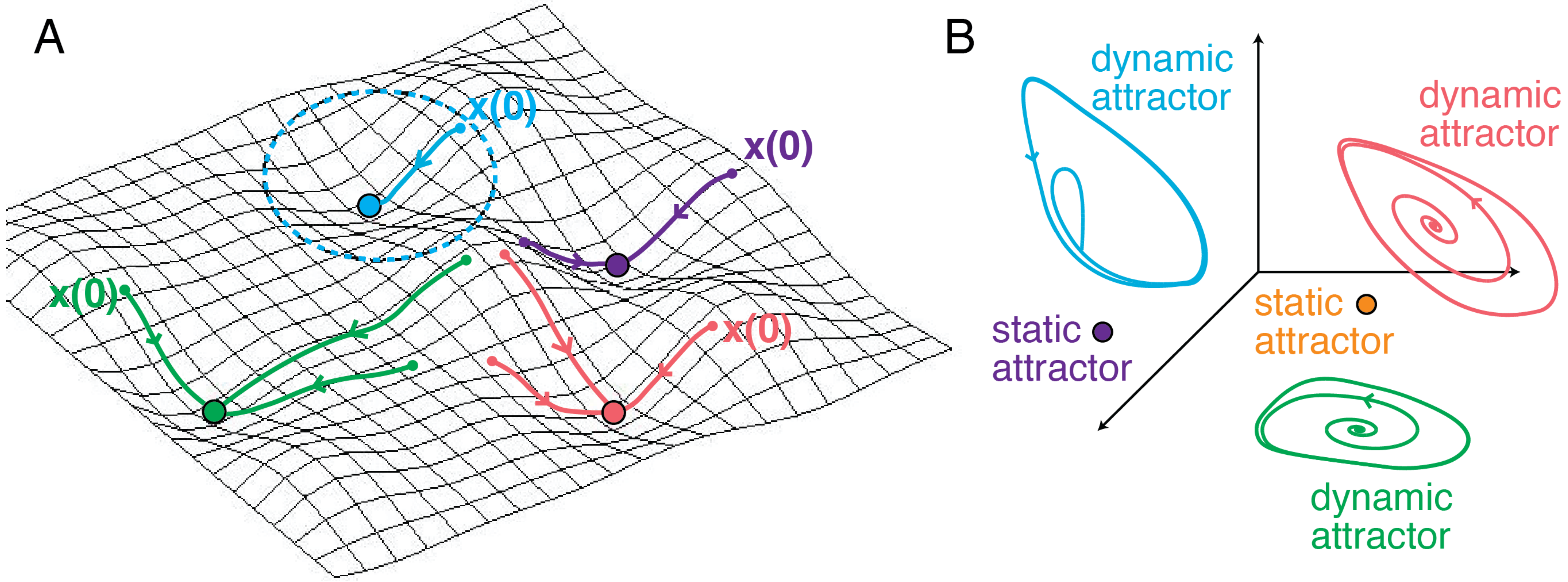




We had many mathematical results, called “graph rules” on CTLNs.

Now many of those results also apply to E-I TLNs built from graphs!





Curto & Morrison, 2023 (review paper): Graph rules for recurrent neural network dynamics

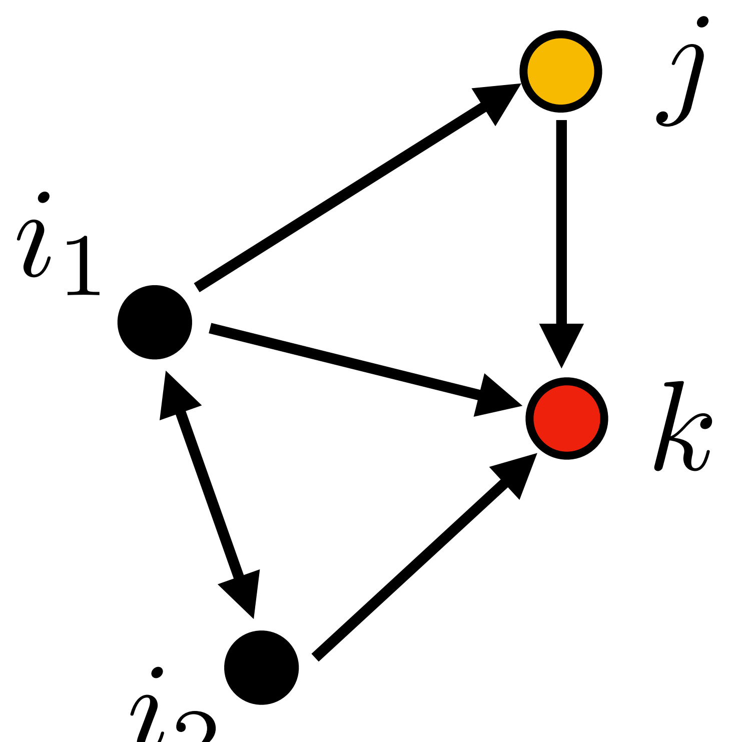
Domination

Definition 1.1. Let $j, k \in [n]$ be vertices of G. We say that k *graphically dominates* j in G if the following two conditions hold:

(i) For each vertex $i \in [n] \setminus \{j, k\}$, if $i \to j$ then $i \to k$.

(ii) $j \to k$ and $k \not\to j$.

If there exists a k that graphically dominates j, we say that j is a *dominated node* (or *dominated vertex*) of G. If G has no dominated nodes, we say that it is *domination free*.



$k > j$

"k dominates j"

"j is a dominated node"

Curto, Geneson, Morrison, 2019

domination is a property of G

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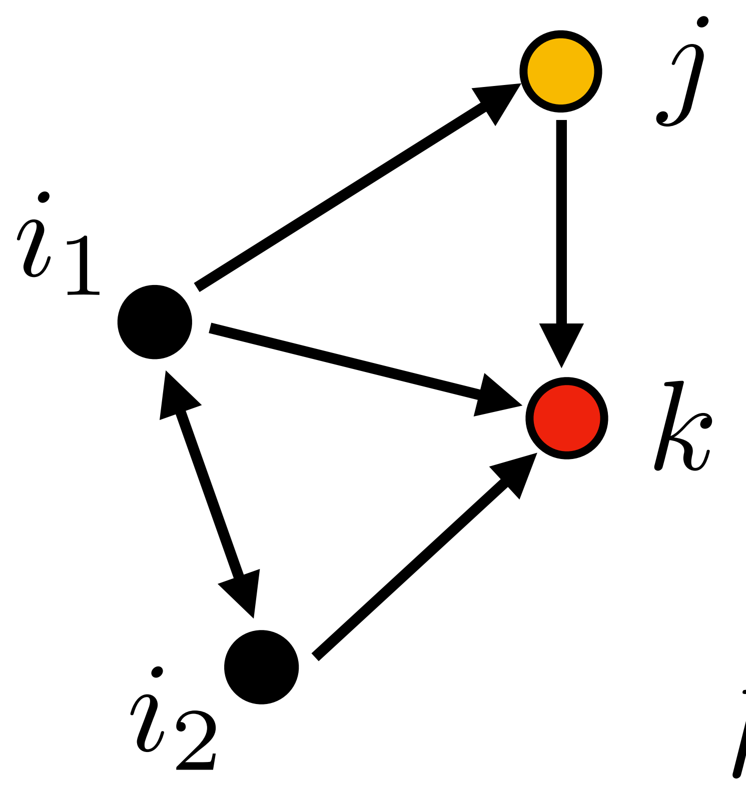

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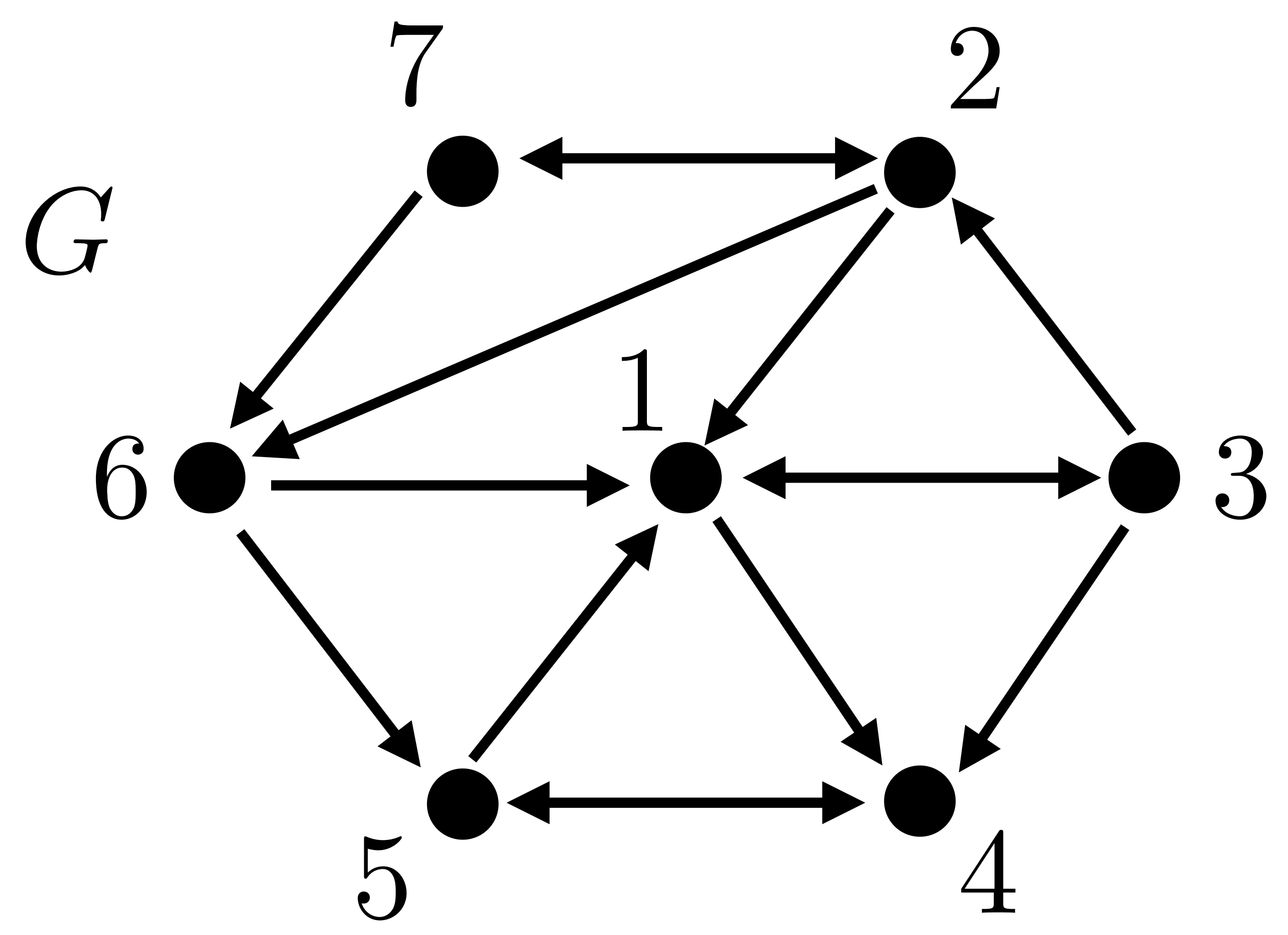
Example



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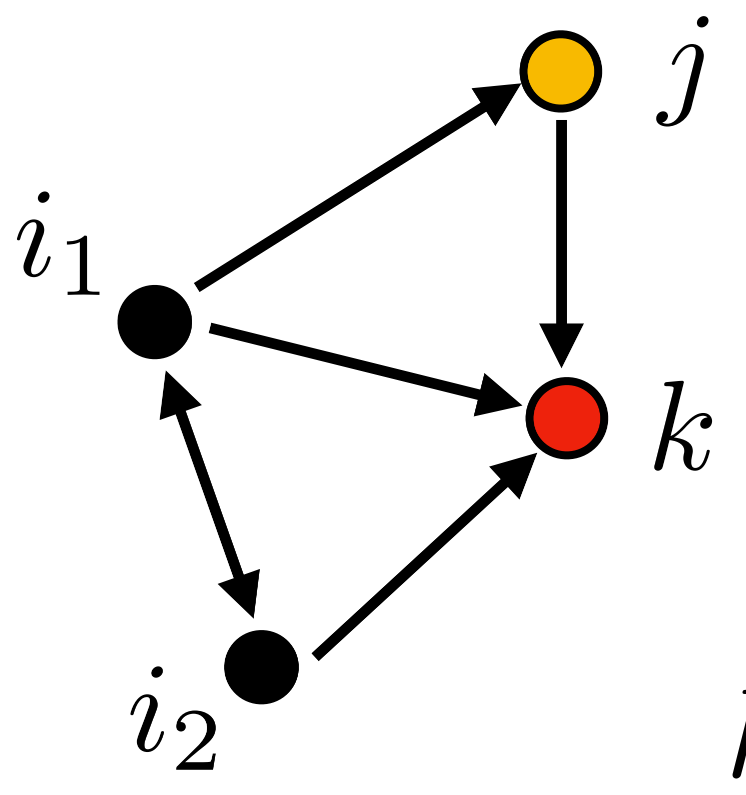

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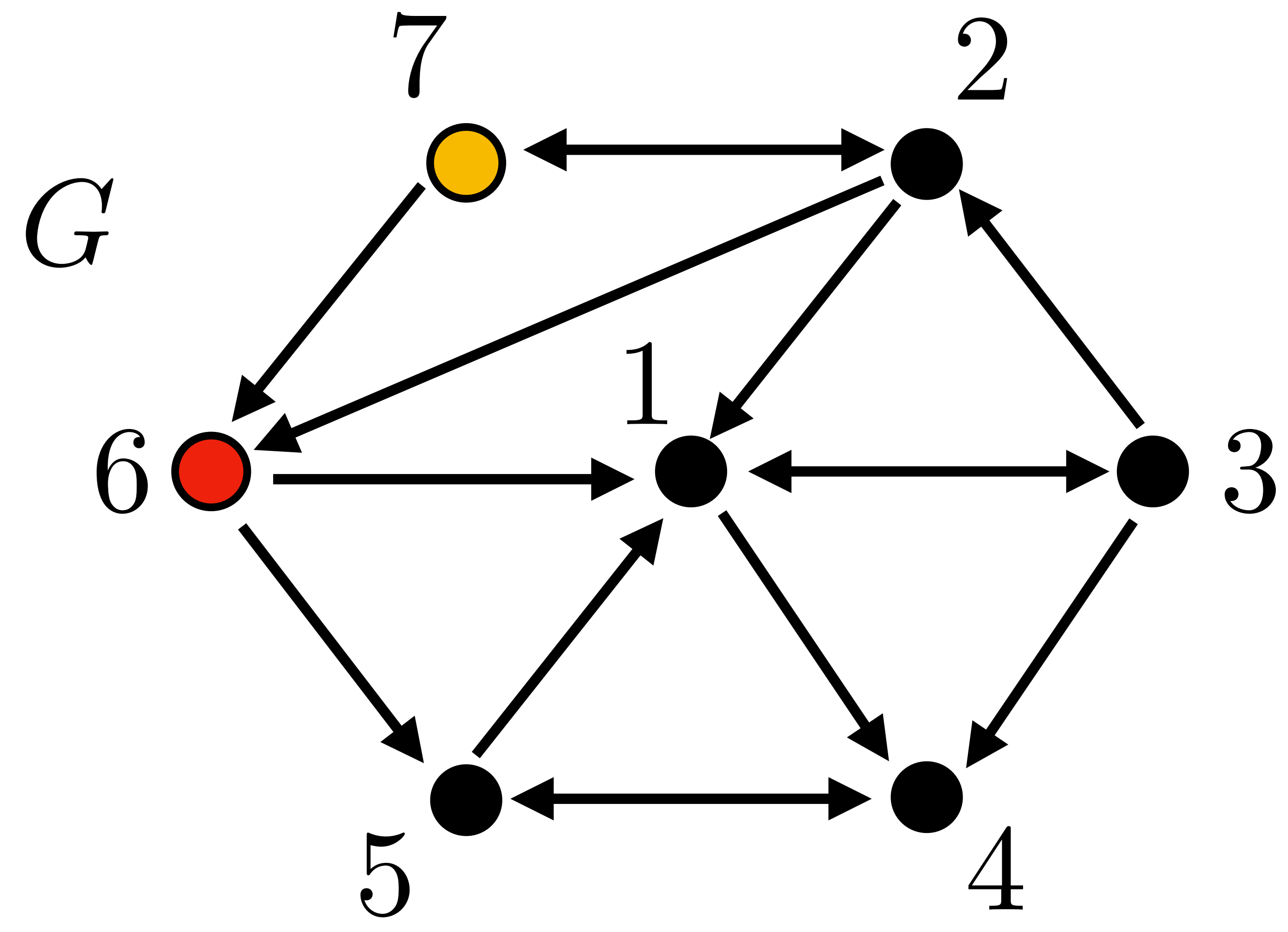

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domination is a property of G

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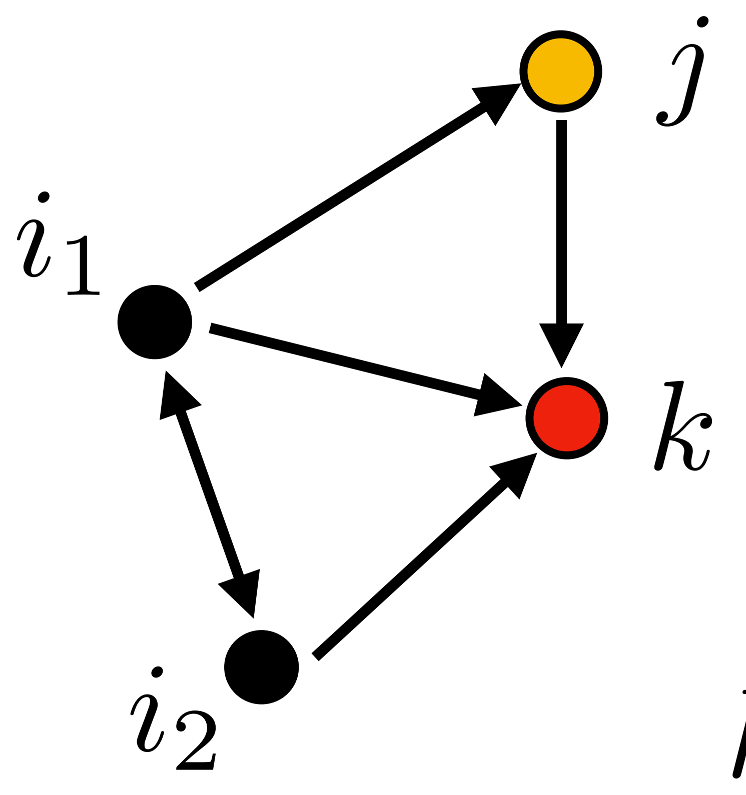

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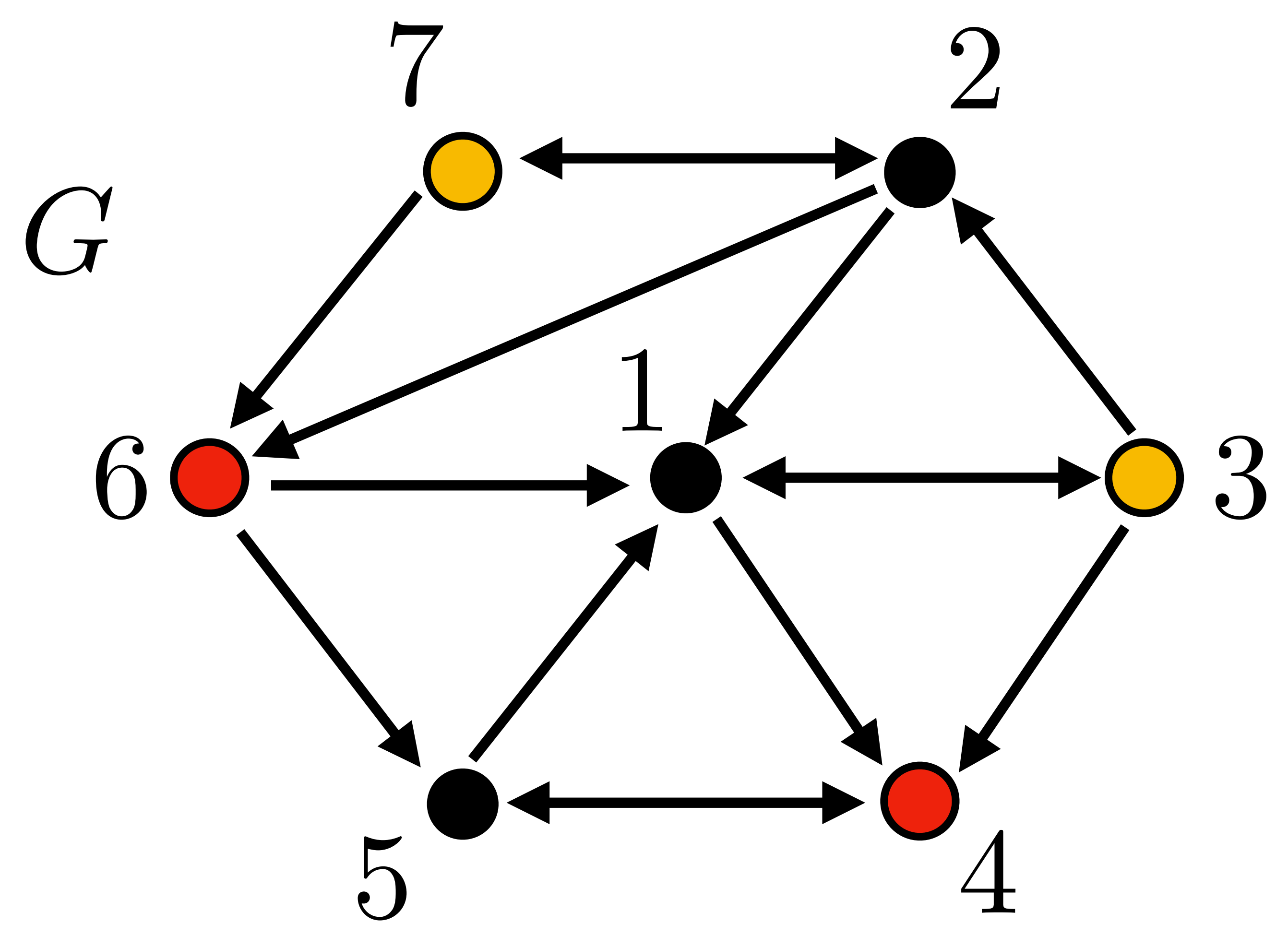


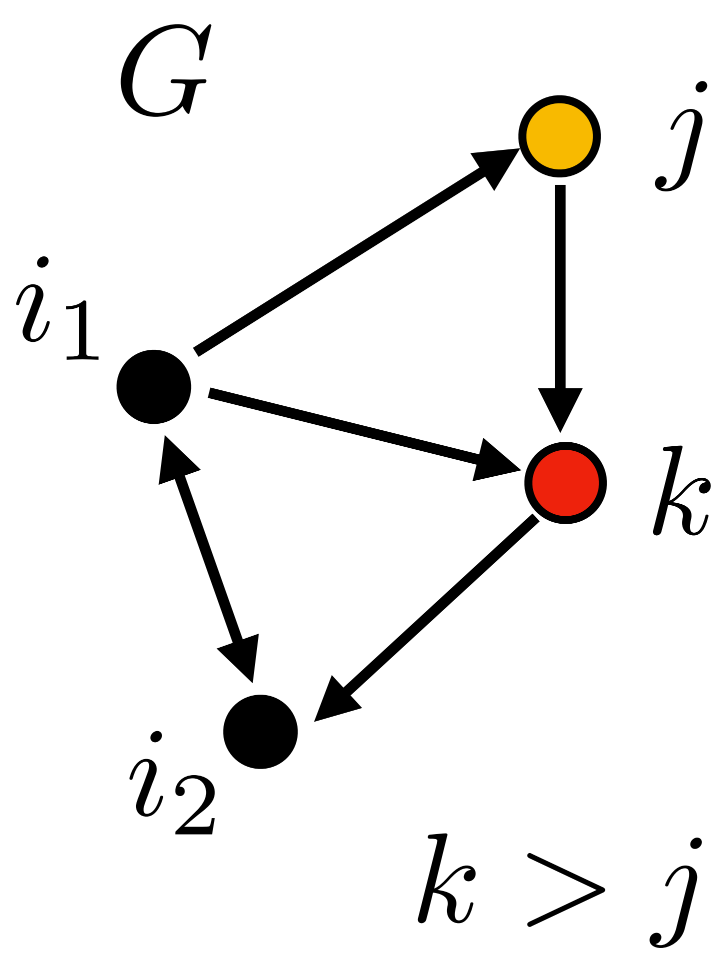
$6 > 7$

$4 > 3$

domination is a property of G

Domination Theorems

Theorem 1 (2024)
If j is a dominated node in G, then it drops out!
I.e., in any gCTLN, we have: $\text{FP}(G) = \text{FP}(G|_{[n]\setminus j})$



Curto 2024 (unpublished)

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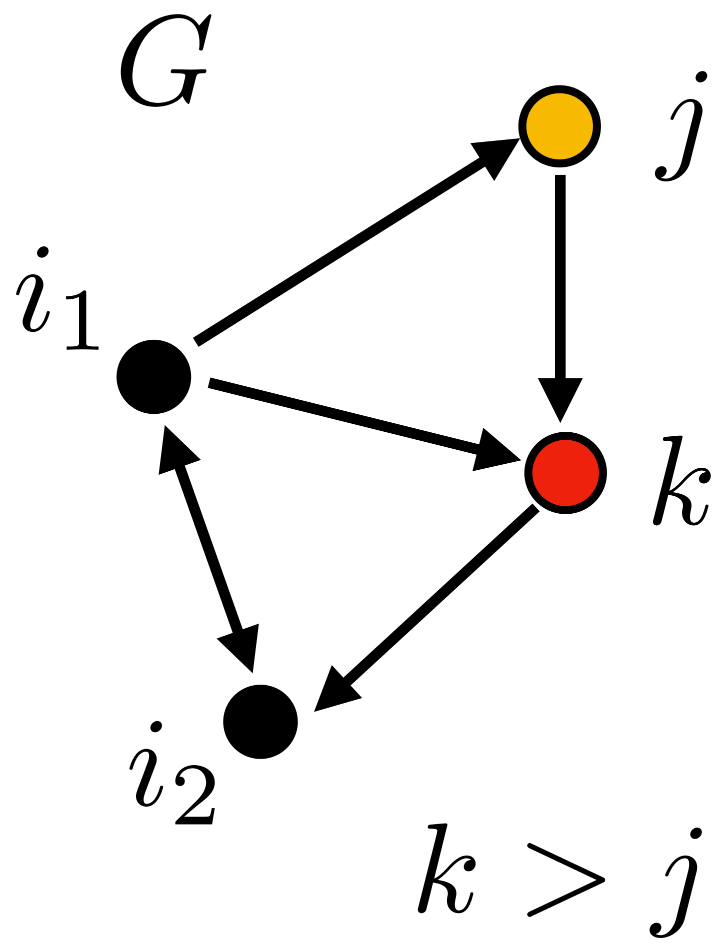

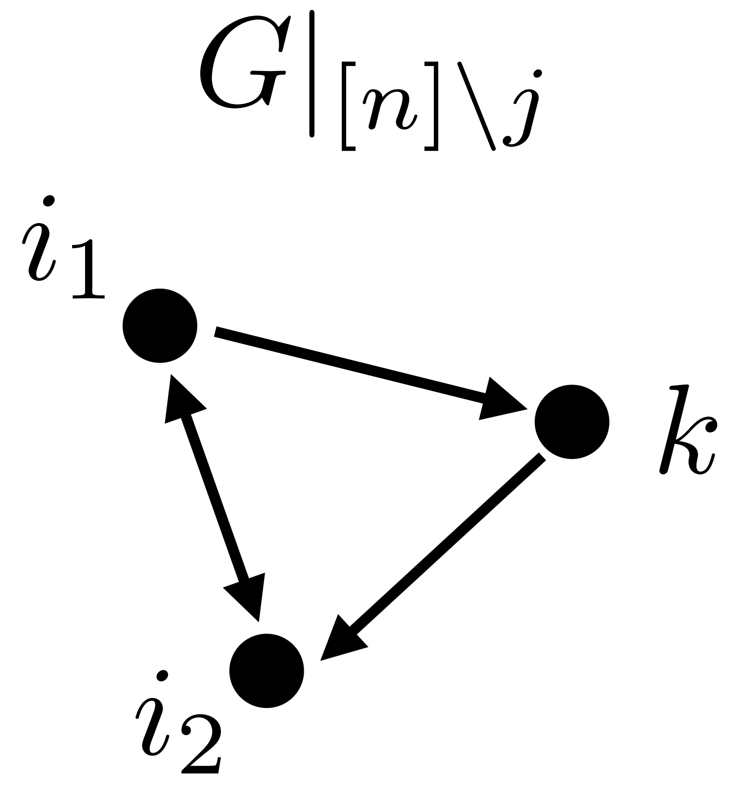

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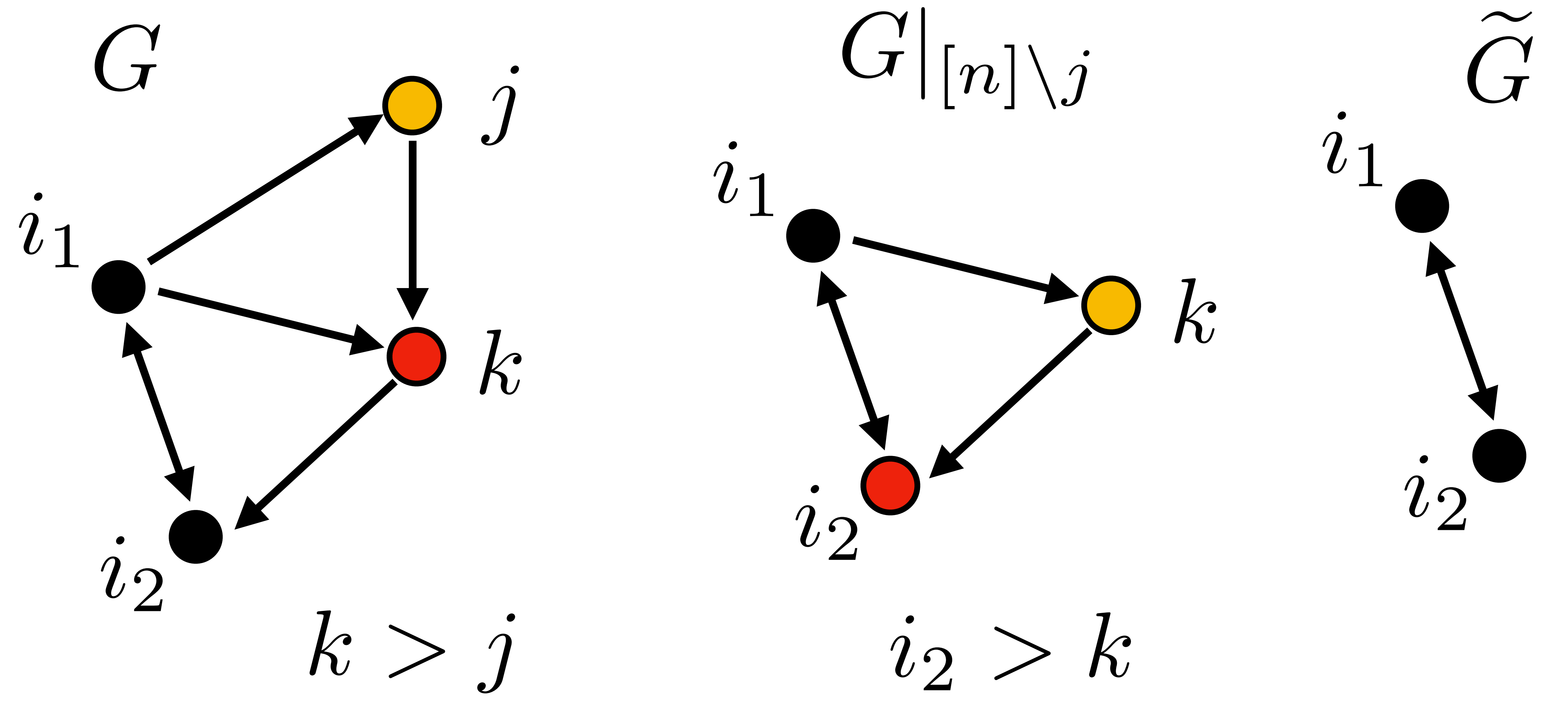
Domination Theorems



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By iteratively removing dominated nodes, the final reduced graph G-tilde is unique. Moreover, $$\mathrm{FP}(G) = \mathrm{FP}(\widetilde{G})$$

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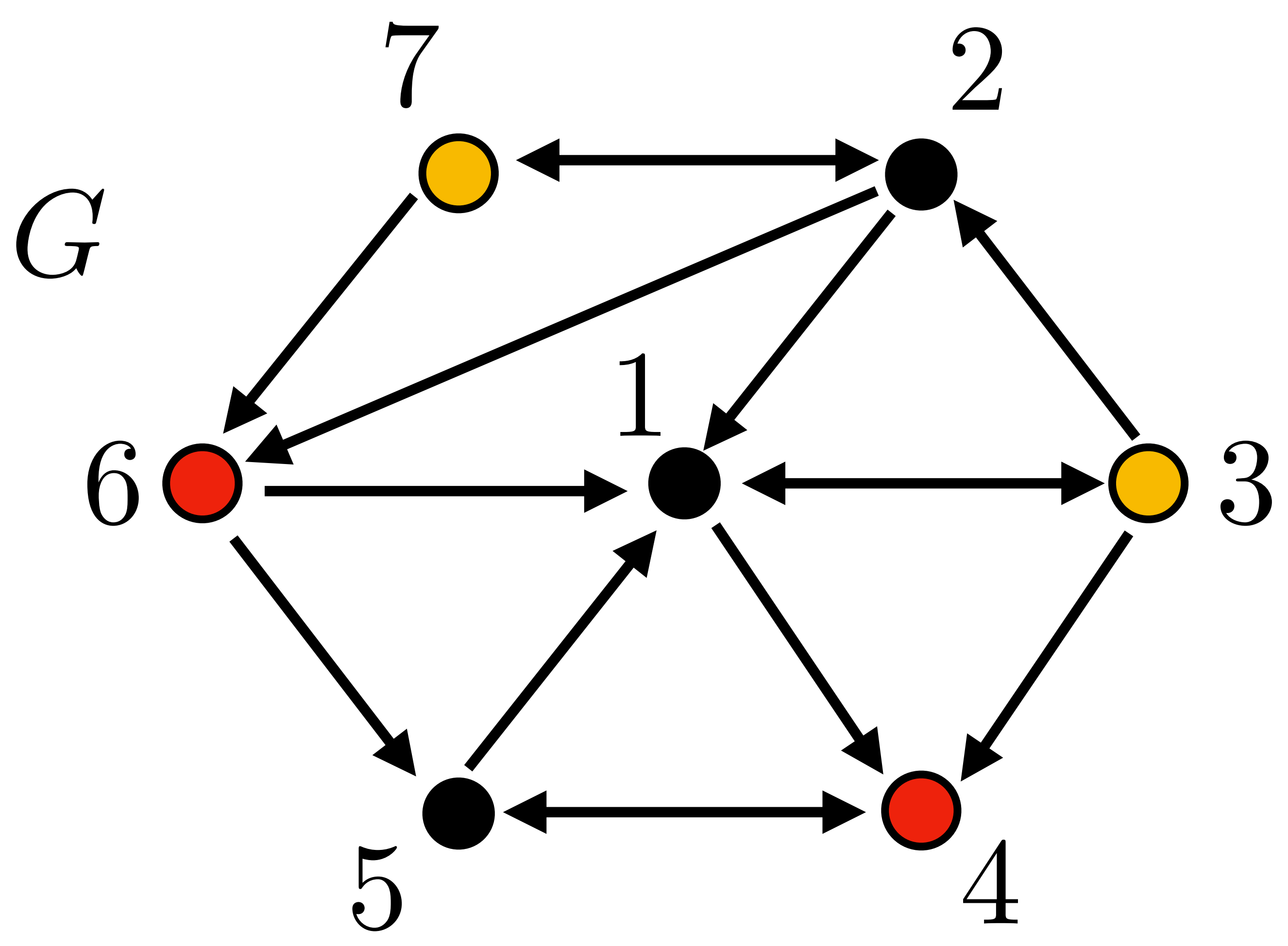

$6 > 7$ $4 > 3$

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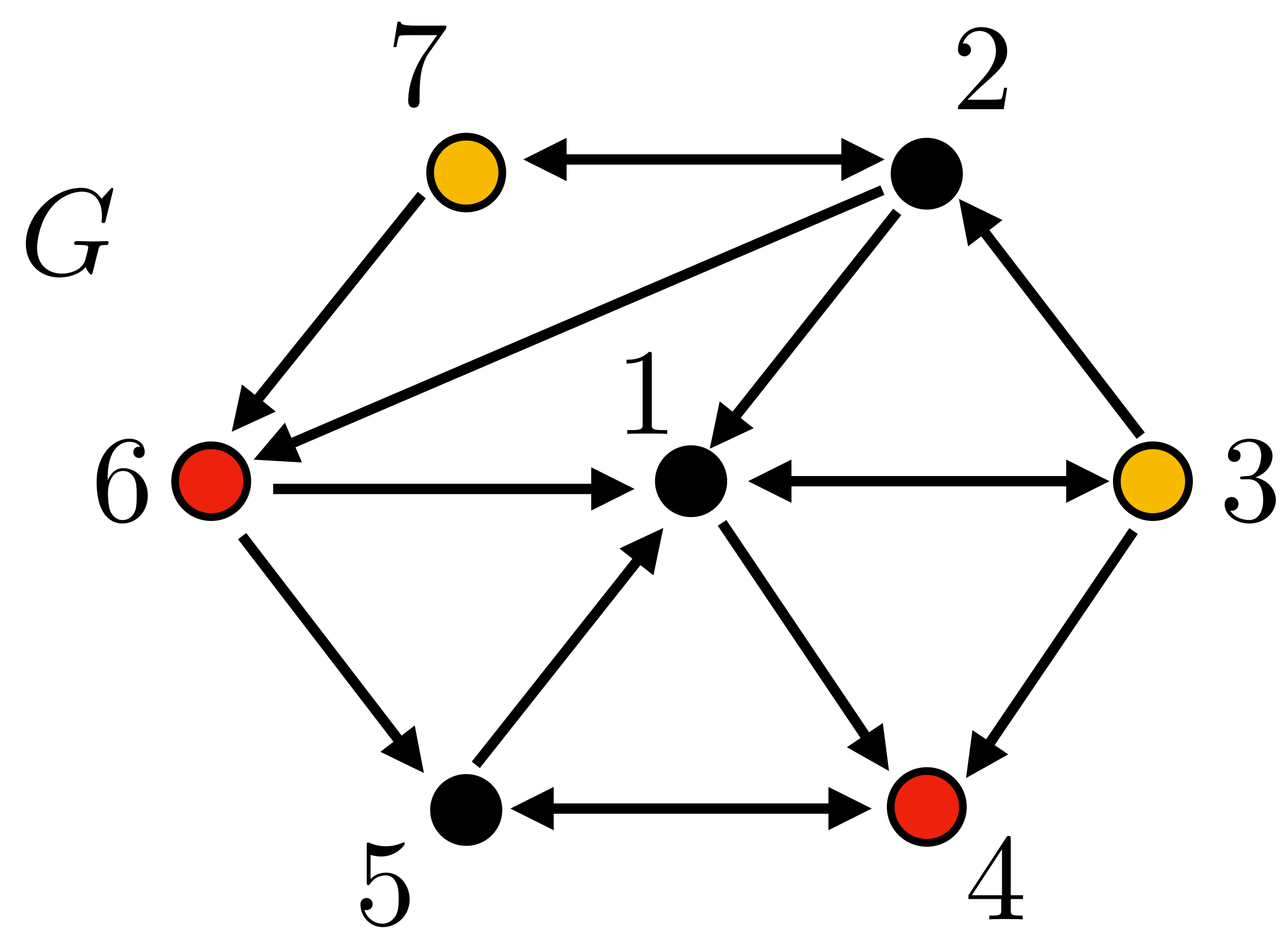

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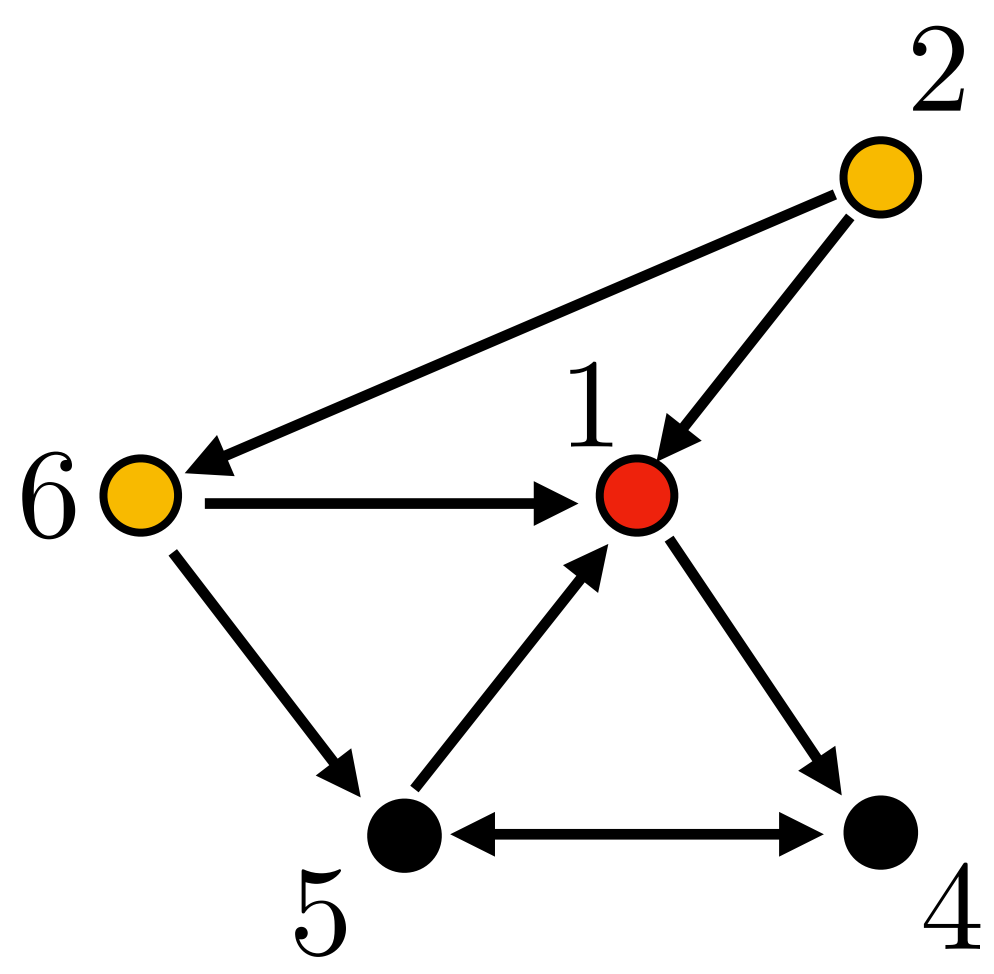

$1 > 2, 6$

Domination Theorems

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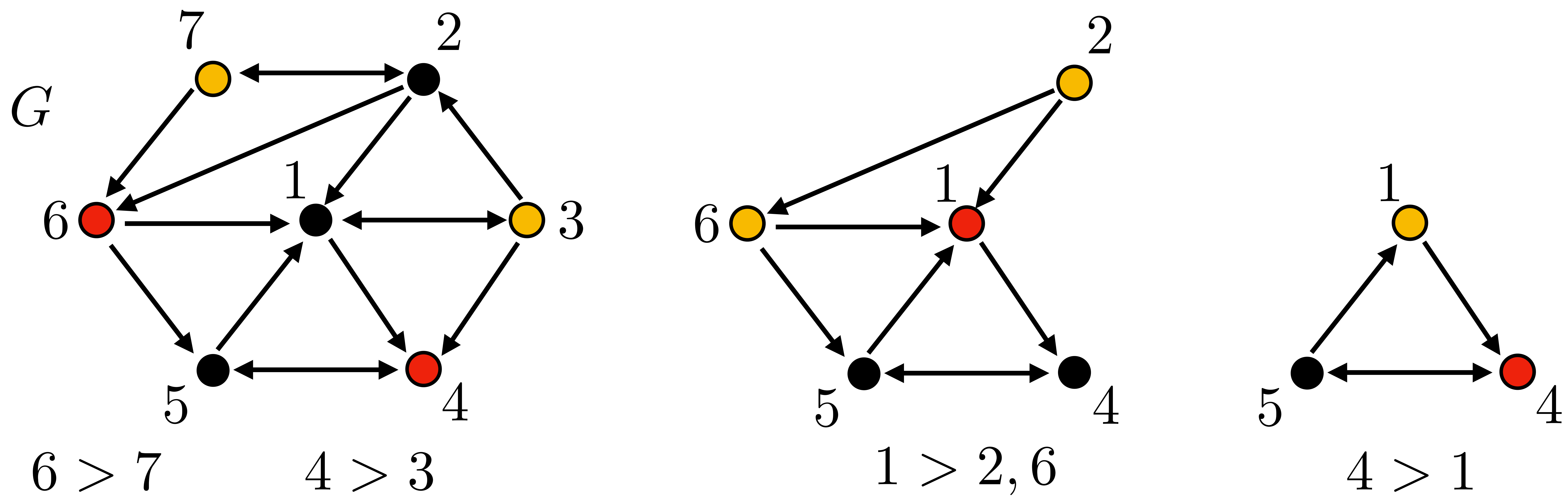

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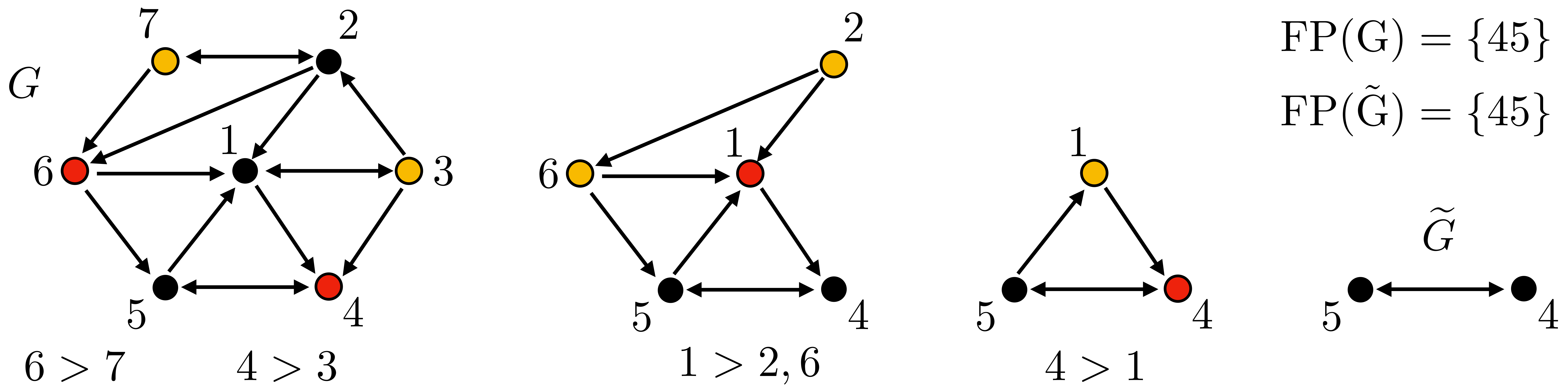
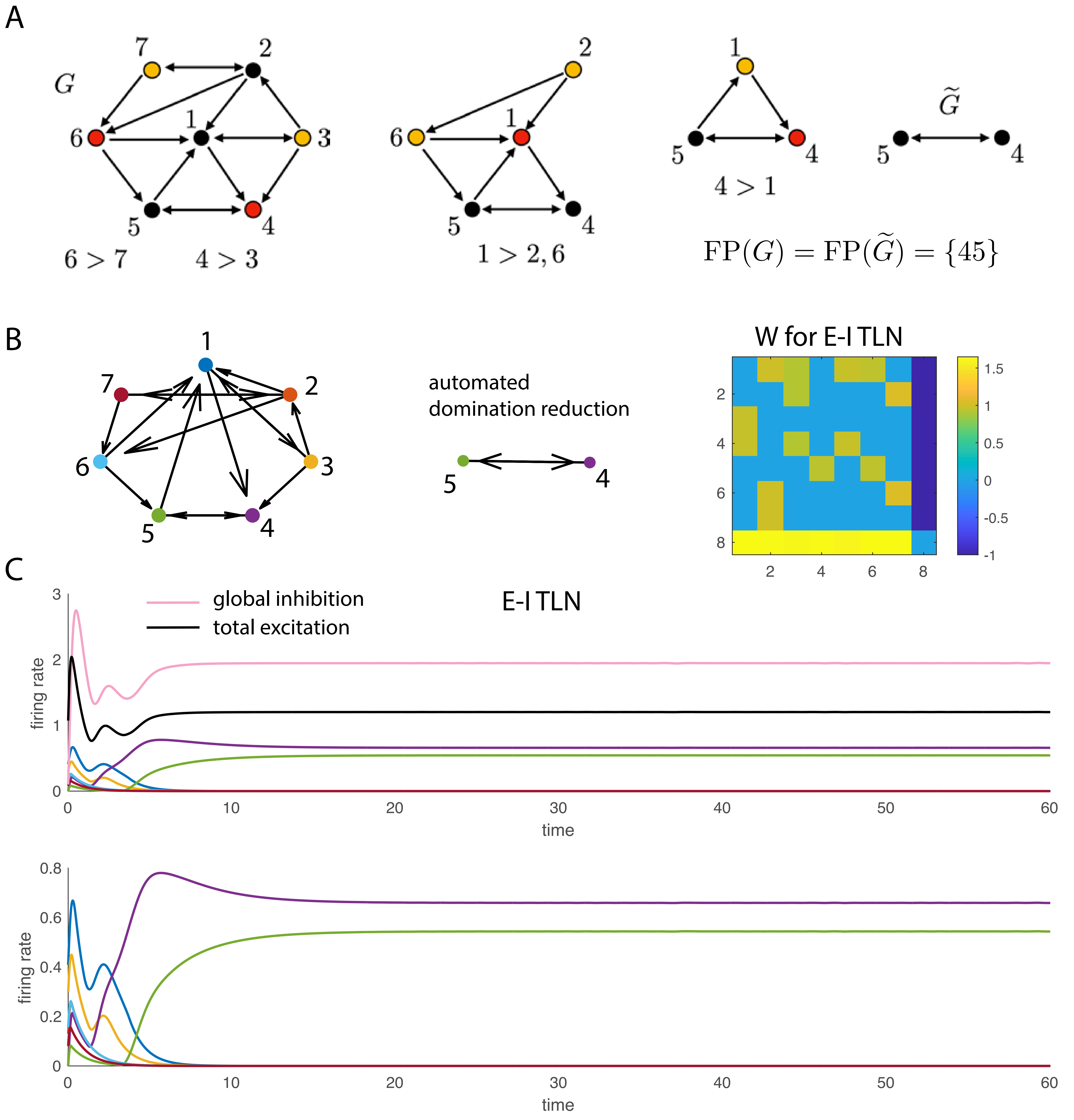
Since E-I TLNs map to gCTLNs with the same fixed points, the domination theorems hold for E-I TLNs, too!

Example





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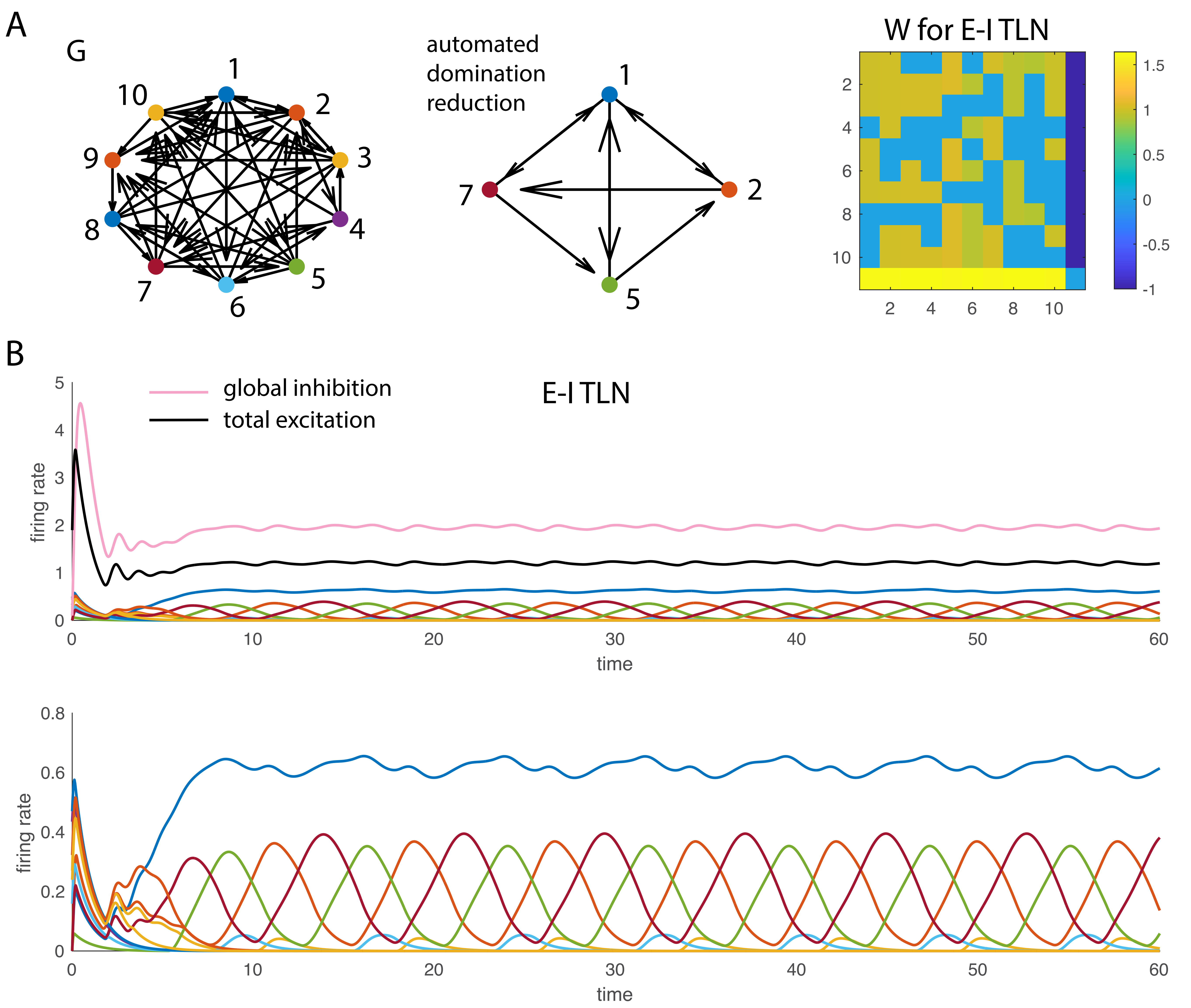


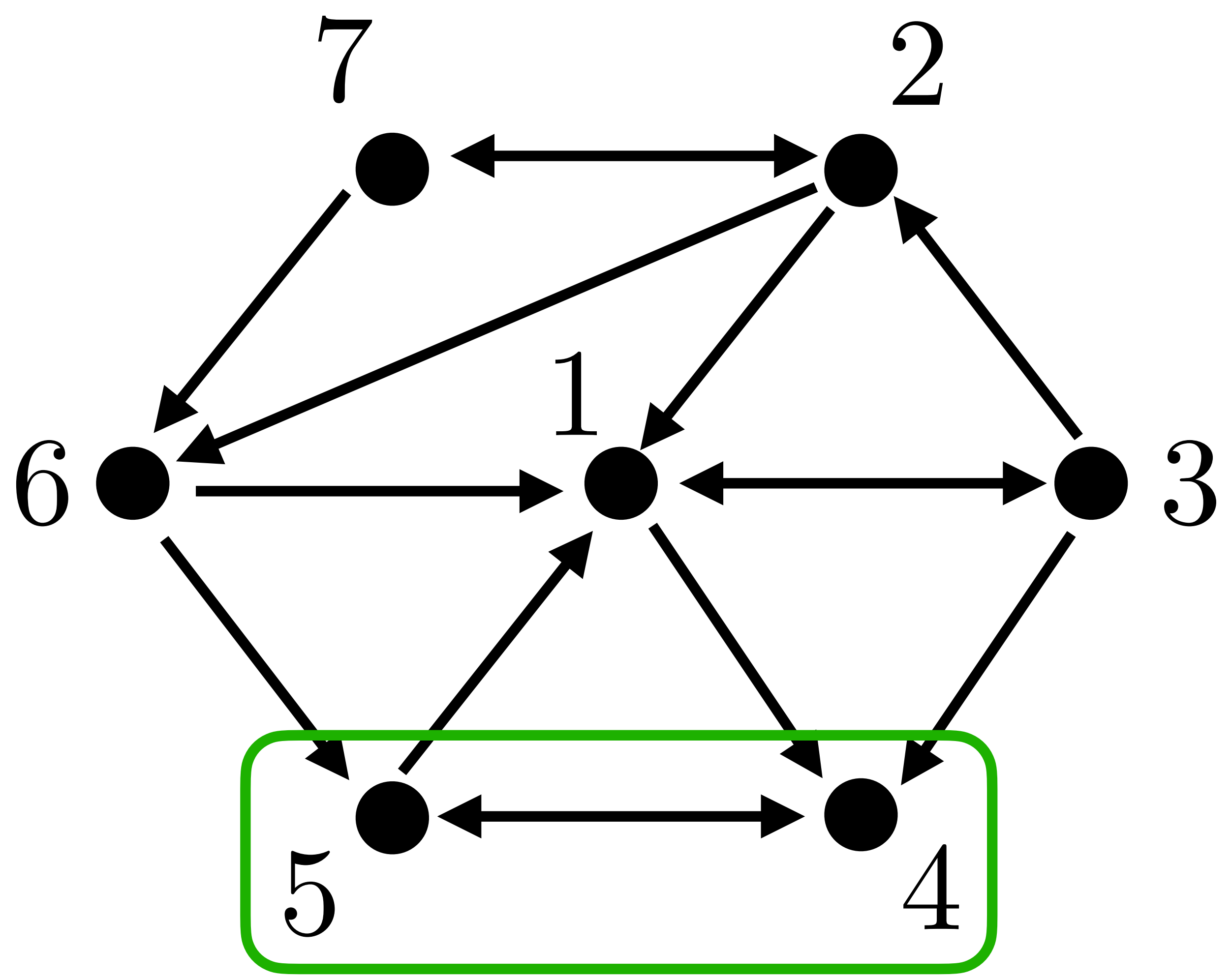
Since E-I TLNs map to gCTLNs with the same fixed points, the domination theorems hold for E-I TLNs, too!

Can domination be useful for connectome analysis?

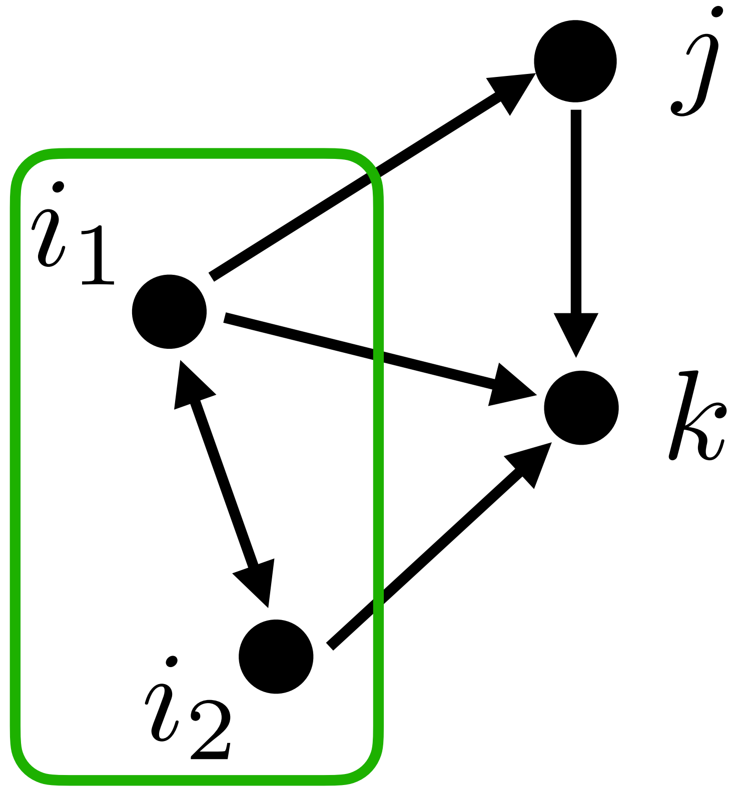
Every graph has a unique domination reduction: $G \longrightarrow \widetilde{G}$

Two graphs with the same reduction are in the same domination equivalence class.

$G \longrightarrow \widetilde{G}$



$H \longrightarrow \widetilde{H}$



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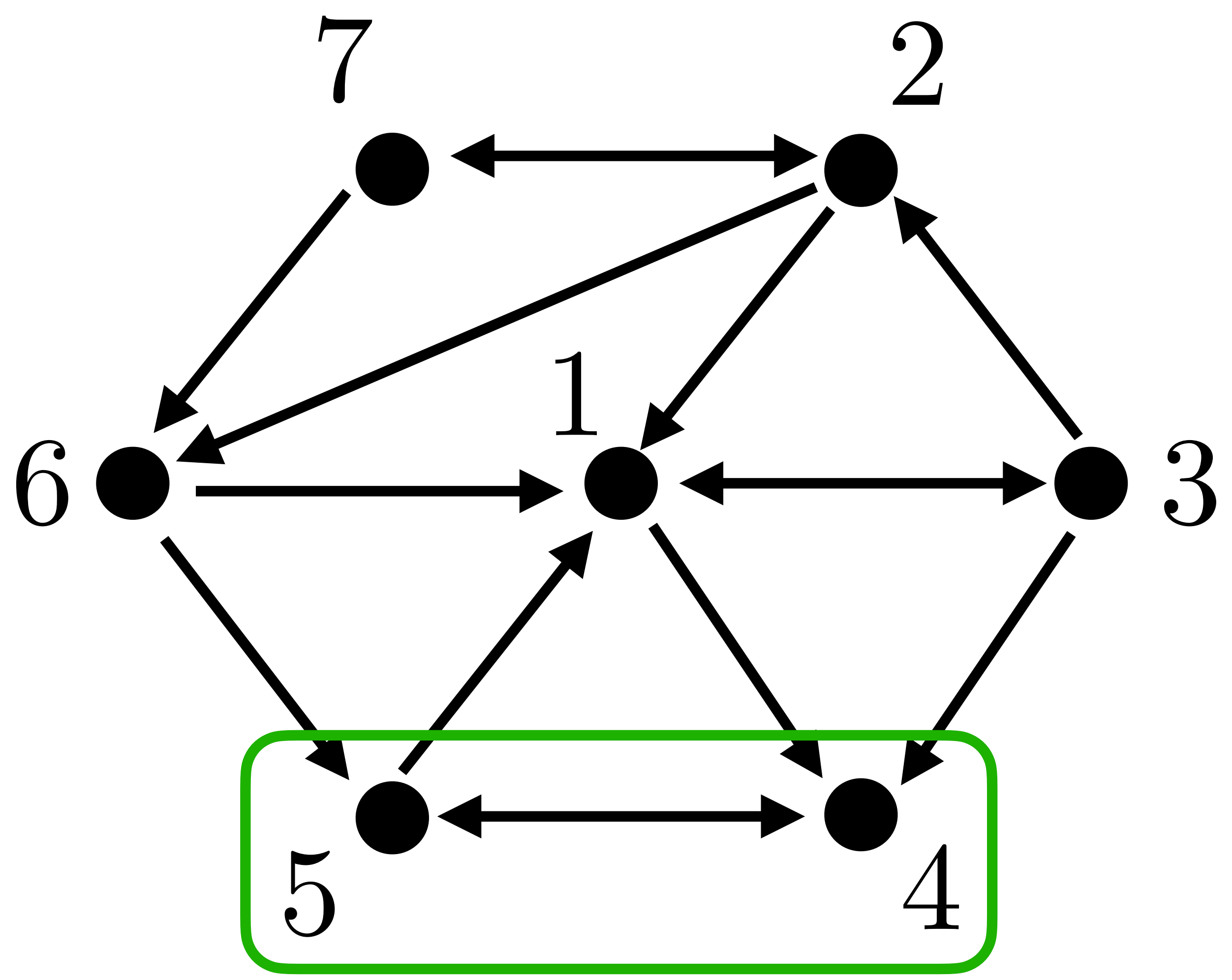

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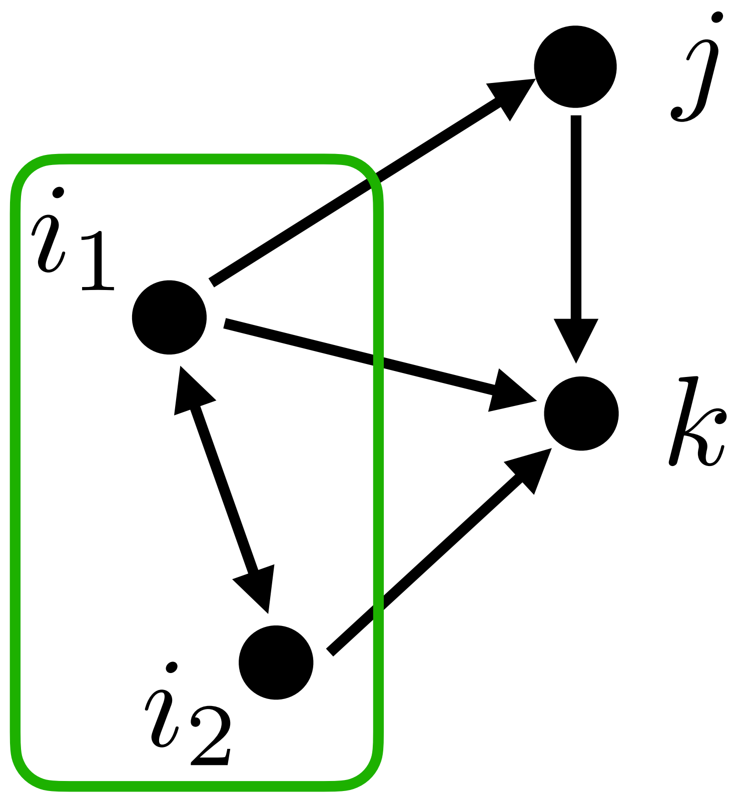


$\widetilde{G} \cong \widetilde{H}$

1. Are overrepresented graphical motifs more likely to be reducible or irreducible?

2. Which motifs are domination-equivalent?

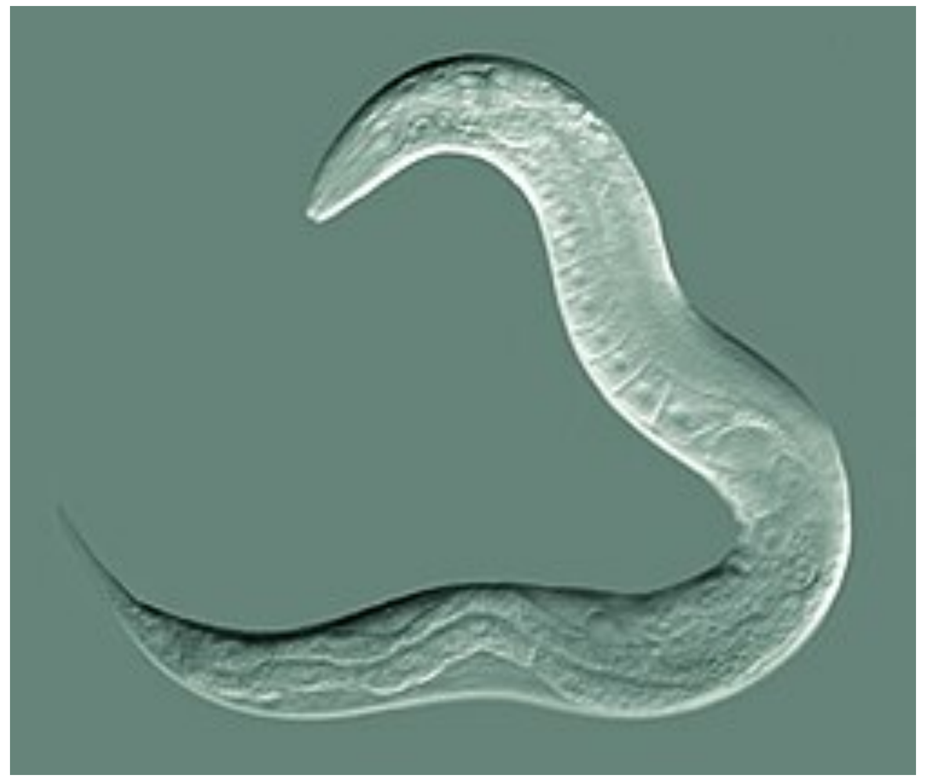
3. What about larger portions of the connectome: do they reduce via domination?

Very preliminary analysis

Graph motifs team at JHU

Jordan Matelsky (also at Penn)
Patricia Rivlin
Michael Robinette
Erik Johnson
Brock Wester

Johns Hopkins University Applied Physics Laboratory,
Research & Exploratory Development Department



C. elegans E-E network:

G has143 nodes

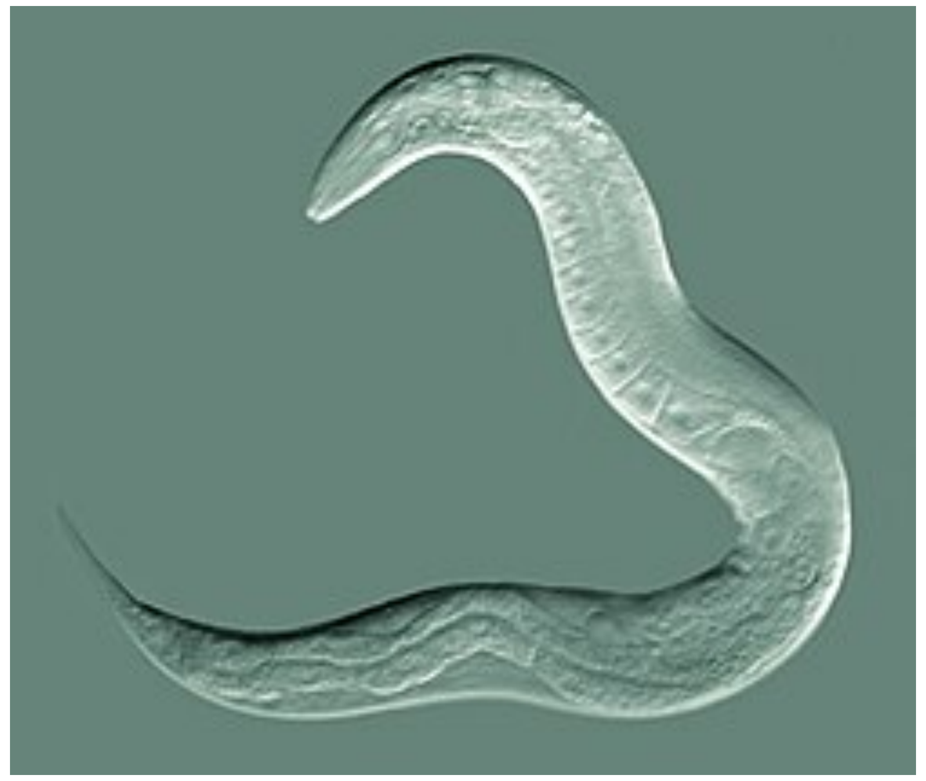
reduced G: 104 nodes

Joaquín Castañeda Castro

We first strip out everything but chemical synapses, then tag neurons by their small-molecule neurotransmitters—acetylcholine/glutamate as excitatory, GABA as inhibitory—next we grab the induced subgraph of neurons that fire ACh/Glu but no GABA. That's our 'excitatory' network. And yes—it's just a conservative, transmitter-based proxy for valence; real C. elegans synaptic polarity is far messier (receptors, modulators, co-transmission, gap junctions, etc.) All blame goes to Jordan Matelsky, Carina did nothing wrong.

Very preliminary analysis

Is a reduction from 143 -> 104 nodes common or rare in a random graph with matching edge probability?



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Joaquín Castañeda Castro

Very preliminary analysis

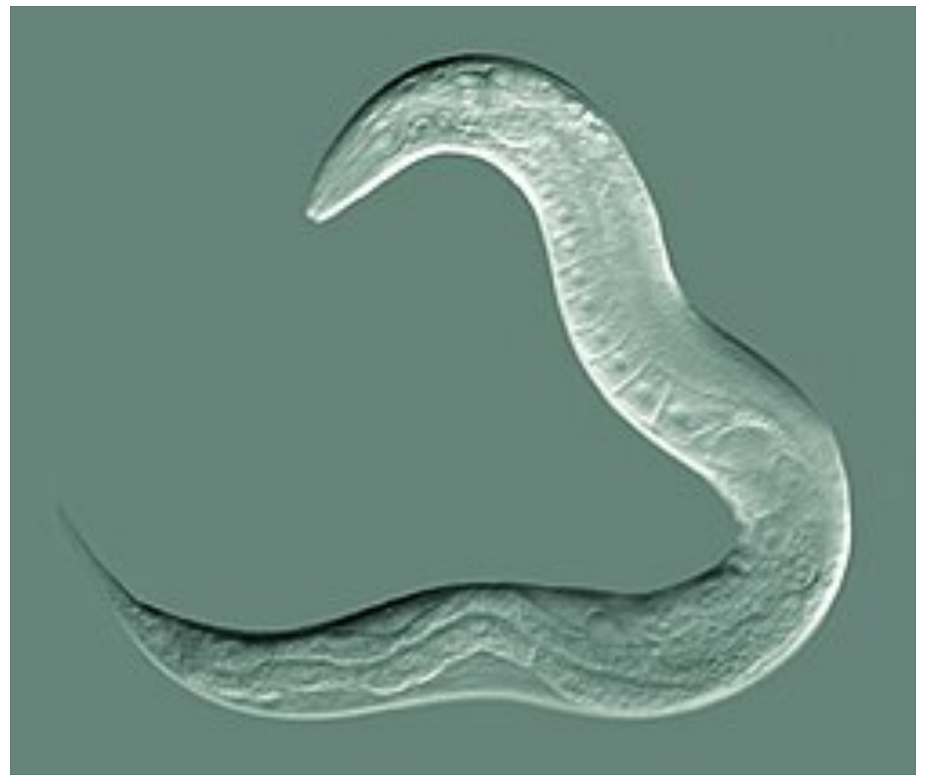
Is a reduction from 143 -> 104 nodes common or rare in a random graph with matching edge probability?

1 million E-R random graphs with matching p = 0.054

Distribution of domination reductions:

- 143 nodes: 782,590
- 142 nodes: 189,951
- 141 nodes: 24,951
- 140 nodes: 2,307
- 139 nodes: 185
- 138 nodes: 15
- 137 nodes: 1

VERY RARE!!



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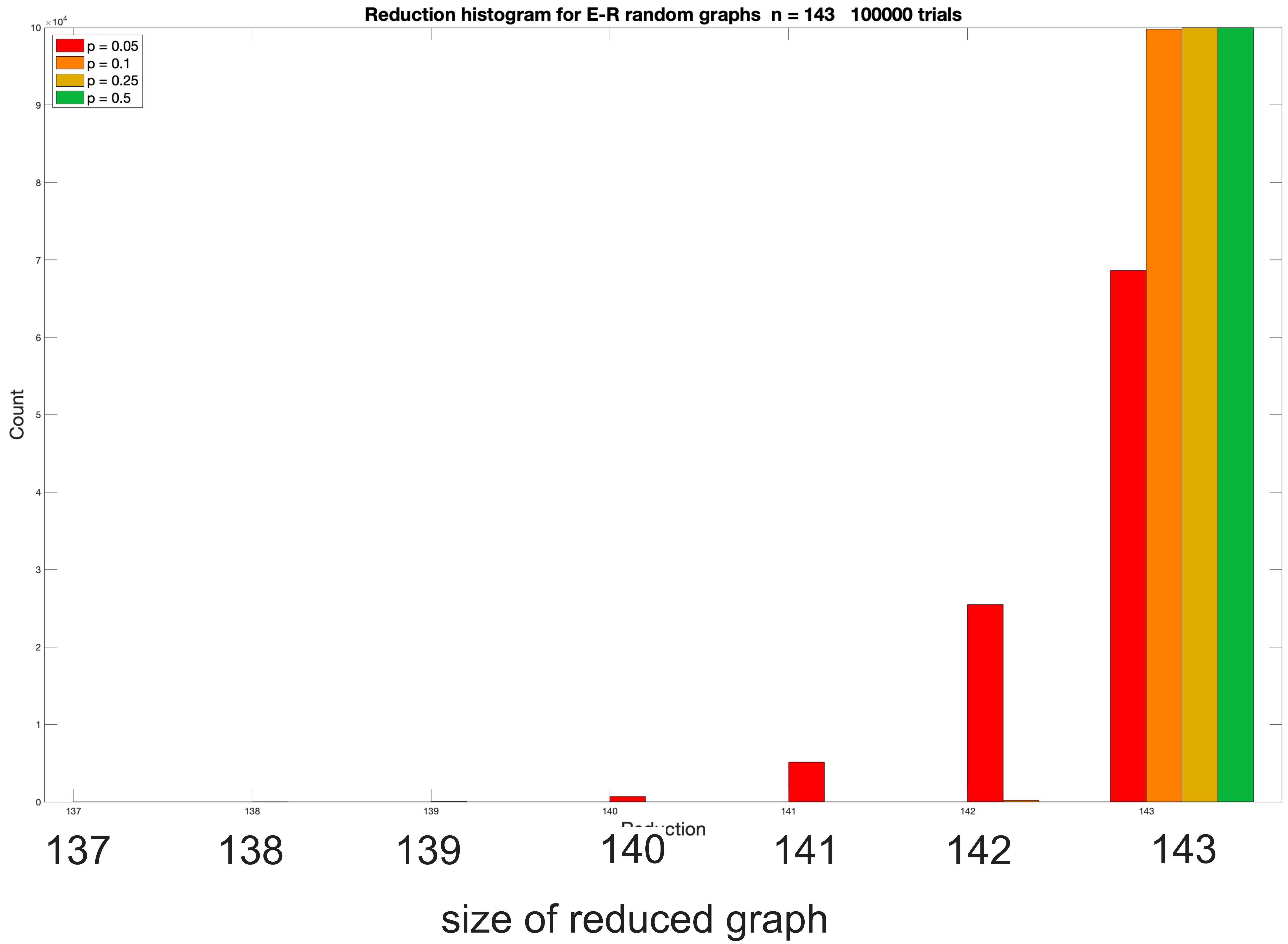
Reduction sizes of E-R random graphs of size n=143 with p = 0.05, 0.1, 0.25, 0.5



Back to our motivating questions and ideas:

1. How does connectivity shape dynamics?

2. The relationship between connectivity and neural activity depends on the dynamical system you associate to the connectome.

3. By studying neuroscience-inspired (nonlinear!) dynamical systems on graphs, we can generate hypotheses about the dynamic meaning/role of various network motifs.

Domination is a graph property that comes out of the nonlinear dynamics, it is not something that graph theorists or network scientists were already paying attention to.

Thank you!


Katie Morrison


Caitlyn Parmelee

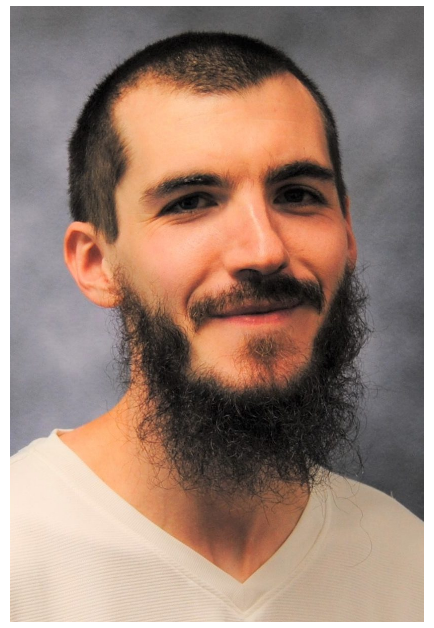

Chris Langdon


Nicole Sanderson


Safaan Sadiq


Jency (Yuchen) Jiang

grad student:
Zelong Li


Jesse Geneson


Caitlin Lienkaemper


Juliana Londoño Alvarez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